LETTERS IN BIOMATHEMATICS
An International Journal

RESEARCH ARTICLE

OPEN ACCESS

Optimal Economic Behavior of Protected Vegetable Farms in Baghdad Province (2025)

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ABSTRACT

This study aims to estimate and analyze the production and cost functions for a sample of protected vegetable farms producing cucumber crops in Baghdad Governorate during the 2025 agricultural season. The analysis is based on examining the relationships between production inputs and outputs in order to determine the level of economic and technical efficiency in the utilization of production resources. Through these functions, it is possible to identify the production stage characterizing protected agriculture in Iraq. In addition, the study seeks to determine both the optimal output level that minimizes production costs and the output level that maximizes profits by analyzing the estimated production and cost functions. The study relies on cross-sectional data collected from a sample of 36 farms located in the outskirts of Baghdad, representing the scale of production resources involved in the production process. The statistical results of estimating the natural logarithmic production function revealed a positive relationship between inputs and outputs. The findings indicate that the farms operate under increasing returns to scale, as the estimated production elasticity reached 1.434, which suggests that protected agriculture in Iraq is still operating within the first stage of production. The cost function was also estimated, and the cubic cost function proved to be the most appropriate model. The results showed that the output level that achieves the minimum average cost amounted to 286 tons of cucumber production for the sampled farms. Meanwhile, the output level that maximizes profit reached 445 tons, whereas the actual production level during the 2025 study season was only 215 tons.

ARTICLE HISTORY
Received September 14, 2026
Accepted October 29, 2026

KEYWORDS
Cucumber Production, Agricultural Economics, Economic Efficiency, Profit Maximization, Optimal Output Level, Cross-Sectional Data.

1. Introduction

Protected vegetable cultivation represents one of the most important agricultural production systems, particularly for crops such as cucumbers, tomatoes, eggplants, peppers, and okra, in addition to several other protected vegetable crops. In this study, special emphasis is placed on the cucumber crop due to its significant economic importance, as it occupies an advanced position among protected vegetable crops in terms of consumption.

Moreover, cucumber cultivation holds a prominent rank with respect to the cultivated area allocated to protected vegetable production under the supervision of the Ministry of Agriculture. The crop is widely marketed within Baghdad Governorate. However, farm productivity varies considerably from one farm to another, primarily due to differences in farmers' efficiency in using production inputs, fluctuations in cultivated areas, variations in soil fertility, and other contributing factors. Accordingly, this research focuses on conducting an economic analysis of the production and cost functions in order to identify both the optimal level of output and the profit-maximizing level of production.

Protected agriculture is considered a relatively modern agricultural practice, and its recent introduction into the field of production has resulted in a lack of reliable information regarding production levels, production costs, and the specific stage of production at which output is generated. In addition, there is insufficient information about the optimal scale of production and the average cost of producing one ton of cucumbers under protected cultivation systems. Furthermore, noticeable variations exist in farm productivity and in the utilization of production inputs, along with a general scarcity of empirical studies addressing protected vegetable cultivation.

2. Research Objectives

1. Estimating the production and cost functions of protected agriculture.
2. Determining the optimal scale of production.

3. Identifying the profit-maximizing level of output and comparing it with the current level of production.

3. Research Methodology and Analytical Approach

The primary data required for this study were collected through a specially designed questionnaire distributed to all plastic greenhouse farmers producing cucumbers. The sample consisted of 36 farms selected from a total population of 55 farms located in the outskirts of Baghdad.

After collecting the data, they were classified and tabulated according to the independent variables involved in the production process. The data were then analyzed using multiple linear regression techniques, and the estimated models were statistically tested to ensure their validity and reliability.

4. Production Function

In this study, several mathematical functional forms were statistically examined, including the linear, quadratic, and logarithmic specifications. Subsequently, the production function of cucumber output was estimated, followed by the estimation of the cost function.

The production function is defined as the technical relationship that links production inputs with the resulting output of goods and services. Several types of production functions can be employed to estimate the relationship between agricultural output and its inputs. In the present research, the natural logarithmic (Cobb–Douglas) production function was adopted and specified as follows (Al-Zubaidi and Yaseen,1998):

$$Y = A \cdot X_1^{\beta^1} \cdot X_2^{\beta^2} \cdot X_3^{\beta^3} \cdot X_4^{\beta^4} \cdot X_5^{\beta^5} \cdot U \quad \text{.....(1)}$$

This model was transformed into its linear logarithmic form to facilitate estimation:

$$\ln Y = \ln a + b_1 \ln X_1 + b_2 \ln X_2 + b_3 \ln X_3 + b_4 \ln X_4 + b_5 \ln X_5 + \ln u \quad \text{.....(2)}$$

Where:

- Y: Quantity of cucumber output (tons per greenhouse).
- X_1: Cultivated area (donum).
- X_2: Capital inputs, including seeds (kg), fertilizers (kg), mechanical labor, manual labor, and pesticides (liters).
- X_3: Hired labor (hours per donum).
- X_4: Family labor (hours per donum).
- X_5: Years of experience in protected agriculture.
- U: Random error term representing the influence of other variables not explicitly included in the model.

5. Cost Function

The cost function represents the relationship between the expenditures incurred by the producer in obtaining the production inputs used to achieve a given level of output. In other words, the cost function expresses production costs as a function of output quantity (Debrtin,1986).

Cost functions may take several functional forms, including linear,

quadratic, and cubic specifications, depending on the nature of production technology and cost behavior (Debrtin,1986).

$Tc = f(Y)$

$Tc = b_0 + b_1 Y$

$Tc = b_0 + b_1 Y + b_2 Y^2$

$Tc = b_0 + b_1 Y + b_2 Y^2 + b_3 Y^3$

6. Results and Discussion

1. Estimation of the Production Function

Several functional forms were examined to estimate the production function. The logarithmic form proved to be the most appropriate specification. The estimated production function is expressed as follows (Koutsoyiammis,1977):

$$\ln Q = 2.13 + 0.14 \ln X_1 + 0.82 \ln X_2 + 0.071 \ln X_3 + 0.31 \ln X_4 + 0.093 \ln X_5$$

t = (2.87) (5.19) (8.21) (3.12) (6.14)

$R^2 = 0.98$ D.W. = 2.28 F = 474.85

7. Statistical Analysis

The statistical estimation results of the cucumber production function indicate that all estimated coefficients are statistically significant at the 1% level, according to the t-test. Moreover, the model as a whole is highly significant based on the F-test. The coefficient of determination (R^2) shows that 98% of the variation in output is explained by changes in the independent variables included in the model, while the remaining 2% is attributed to other factors not accounted for within the estimated model (Raad *et al.,* 2008).

Econometric Analysis

For the estimated model to be considered reliable and valid in explaining the studied phenomenon, it is necessary to conduct diagnostic econometric tests related to multicollinearity, autocorrelation, and heteroskedasticity. These tests were performed, and the results indicated that the model is free from major econometric problems.

The Durbin–Watson (D.W) test confirmed the absence of autocorrelation, as the calculated value of D = 2.28, which lies between the lower and upper critical bounds:

$1.65<d<2.35$

$1.65<2.28<2.35$

This result indicates that the model does not suffer from serial correlation at the 1% significance level, thereby confirming the econometric soundness of the estimated production function (Raad *et al.,* 2022).

Econometric Diagnostics and Economic Interpretation

The Park test results indicate the absence of a heteroskedasticity problem. This test involved estimating a regression equation in which the squared residuals were treated as the dependent variable, while cucumber output was included as the independent variable. The estimated relationship is expressed as follows (Dowling,1985):

$\text{Ln } e_i^2 = 5.3 - 0.27 \text{ Ln } Y^2$

$t = (0.12)\ (-0.089)$

$F = 0.73,\ R^2 = 0.16$

These results confirm that the variance of the error term is constant, indicating that the estimated model does not suffer from heteroskedasticity.

Economic Analysis

The estimated production function for cucumber output shows that the signs of all coefficients are consistent with economic theory. In the logarithmic specification, the estimated coefficients represent output elasticities with respect to the corresponding inputs (Lagard and Walters,1978).

The results also reveal that the total production elasticity, calculated as the sum of individual input elasticities, equals 1.434, which is greater than unity. This indicates the presence of increasing returns to scale, implying that an increase in all production inputs by 1% leads to an increase in output by approximately 1.434%.

This finding suggests that protected vegetable production in Iraq has not yet reached the economically efficient stage of production and remains within the first stage of production, characterized by increasing returns to scale.

Furthermore, the elasticity of the capital input (X_2) exceeds half of the total elasticity, highlighting the capital-intensive nature of protected agriculture and indicating a strong dependence on capital inputs in this production system (Al-Jumaily, 2007).

2. Estimation of the Cost Function

In estimating the cost function, the quantity of cucumber output per donum (Y, measured in kilograms) was considered the explanatory variable, while total cost (TC), measured in Iraqi dinars per donum, was treated as the dependent variable.

Three alternative functional forms of the cost function—linear, quadratic, and cubic—were estimated. Based on statistical and econometric criteria, the cubic cost function was found to be the most appropriate specifiation, as shown below (Henderson and Quandt, 1980):

$Tc = -22.172 + 184.105Y - 1.362Y^2 + 0.00238Y^3$

$t = (-1.02)\quad (5.677)\quad (-2.99)\quad (1.77)$

$R^2 = 0.972\quad D.W. = 1.81\quad F = 957.65$

The high coefficient of determination indicates that the model explains approximately 97.2% of the variation in total production costs, while the Durbin–Watson statistic confirms the absence of autocorrelation in the residuals. The F-statistic further demonstrates the overall statistical significance of the estimated cost function.

Second: Determining the Optimal Behavior of the Farmer:

The producer's objective is:

a) Cost minimization.
b) Profit maximization.

***a) Cost Minimization*:**
In the short run, the farmer aims to minimize costs by satisfying two conditions.
The necessary condition is achieved by setting the first derivative of the average cost equal to zero (Al-Hassani, 2006): $\frac{d\ Avc}{d\ Y} = 0$

The sufficient condition requires that the second derivative of the average cost be positive: $\frac{d^2\ Avc}{d\ Y^2} > 0$

From the following cost function, both conditions can be derived:
Tc = -22.172 + 184.105 Y - 1.362 Y^2 + 0.00238 Y^3
Tvc = 184.105 Y - 1.362 Y^2 + 0.00238 Y^3
Avc = $\frac{Tvc}{Y}$ = 184.105 - 1.362 Y + 0.00238 Y2
To determine the output level that minimizes the average variable cost:
$\frac{d\ Avc}{d\ Y}$ = -1.362 + 0.00476 Y = 0
∴ Y^= 286.13
$\frac{d^2\ Avc}{d\ Y^2}$ = 0.00476>0
Thus, the output level at which the lowest average variable cost is achieved is
286.13 tons for the sampled farm.

b) Profit Maximization:

To determine the output level that achieves maximum profit, two conditions must be satisfied (Al-Atabi, 2002):
the necessary condition and the sufficient condition for the profit function.
π= TR - Tc
The necessary condition requires that marginal revenue equal marginal cost:
$\frac{d\ \pi}{d\ Y}$ = MR - MC = 0
MR = MC
The sufficient condition requires that the second derivative of the profit function be negative: $\frac{d2\ \pi}{d\ Y2}$ <0
Given the profit function:
π = TR - Tc
π = P.Y - Tc
= P.Y + 22.172 - 184.105 Y + 1.362 Y^2 - 0.00238 Y^3 =0
$\frac{d\ \pi}{d\ Y}$ = P - 184.105 + 2.724 Y - 0.00714 Y^2 = 0
Substituting the output price P = mp cost 400 dinars per kilogram yields:
400 - 184.105 + 2.724 Y - 0.00714 Y^2 = 0
215.895 + 2.724 Y - 0.00714 Y^2 = 0
Applying the quadratic formula to solve for Y: Y = $\frac{-2.724 \pm\sqrt{7.42+6.16}}{-0.0143}$
∴ Y = 445.7
This quantity satisfies the necessary condition for an optimum Verifying the sufficient condition: $\frac{d2\ \pi}{d\ Y2}$ = 2.724 - 0.01428 Y

= 2.724 - 0.02856 (445.7)
= -10.006<0

Thus, the profit-maximizing output level is 445.7 tons for the sampled farm. Substituting this quantity into the profit function yields the maximum achievable profit of 156.102 million dinars for the sampled farm, averaging approximately 346.89 thousand dinars ton^{-1} per farm.

In contrast, the survey data indicate that the actual production level did not exceed 215 tons for the sampled farm, with an average profit of 220 thousand dinars ton^{-1}.

8. Results

1. The output level that achieves the lowest average cost was determined to be 286 tons of cucumber production.
2. The output level that maximizes profit was found to be 445 tons for the sampled farms. However, the actual production during 2025 was only 215 tons.
3. The study also indicated that the production elasticity reached 1.434, suggesting that protected agriculture in Iraq is still in the

first stage of production.

9. Conclusions

This study concludes that providing highly efficient production resources, particularly seeds, can increase yield per unit area. Production elasticity indicates that output remains high in the early stages of the production process. Therefore, the study recommends adopting a production strategy to achieve optimal resource utilization and move towards rational production. The focus should be on increasing production efficiency by reducing input costs through "optimal use." Simultaneously, subsidized prices should be provided to encourage farmers to utilize resources effectively to achieve profit-maximizing production levels.

Research Overview

* Research Title: Optimal Economic Behavior of Protected Vegetable Farms in Baghdad Province (2025).

* Study Area: The outskirts of Baghdad Governorate, Iraq.

* Study Sample: The research is based on cross-sectional data collected from a sample of 36 plastic greenhouse farms producing cucumber crops.

Authors' Contribution

* Raad Idan Obaid: Conceptualized the research idea, designed the analytical methodology, and performed the estimation of the natural logarithmic production and cubic cost functions.

* Rashid Bani Shannan: Conducted the field survey, collected cross-sectional data from the 36 sampled farms in the outskirts of Baghdad, and handled data classification and tabulation.

* Mohammed Khaleel Jasim: Performed the econometric diagnostic tests (Durbin-Watson and Park tests), interpreted the economic results regarding production elasticity (1.434), and formulated the final recommendations.

Generative AI and AI-Assisted Technology Statement:

The authors (Raad Idan Obaid, Rashid Bani Shannan, and Mohammed Khaleel Jasim) declare that no generative artificial intelligence or AI-assisted technologies were utilized in the preparation, data analysis, or writing of this manuscript. All statistical estimations of the production and cost functions, as well as the determination of the optimal output levels (286 tons and 445 tons), were conducted independently by the researchers using standard econometric techniques.

Conflict of Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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