

Breakpoint Adaptive Neural Fuzzy Inference Regression Model for Predicting the Risk of Cardiovascular Diseases Complications

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ABSTRACT

In this paper, the segmented and breakpoint regression models were estimated using the Ordinary Least Squares (OLS) method and the Adaptive Neuro fuzzy Inference model (ANFIS) under a dataset of (200) patients. The dependent variable (y) cardiovascular complications, six explanatory variables (age, body mass index, blood glucose level, systolic blood pressure, cholesterol level, and C-reactive protein). The Chow test was first performed to identify regression points, revealing two points (110 and 160). The ANFIS model proved superior to the Ordinary Least Squares (OLS) method in estimating the risk of complications. Furthermore, these systems provided accurate estimates that accurately reflected the studied phenomenon, unlike the OLS method, which yielded inaccurate and unreliable estimates. The ANFIS estimates also proved that all research variables were significant and had an impact on the occurrence of complications resulting from cardiovascular disease.

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1. Introduction

In the last several decades, the world has seen a lot of progress in many areas of knowledge, especially in statistics and data analysis. This is because the amount, variety, and complexity of data have grown a lot. This change has made it necessary to create more flexible analytical models that can show how variables really relate to each other, particularly when the data is imprecise, unclear, or ambiguous. Standard statistical models can't explain many complicated events anymore. This has led academics to create hybrid models that use both statistics and artificial intelligence. One of the most important statistical approaches for looking at the link between a dependent variable and one or more independent variables is regression analysis. The goal is to explain this connection and use the independent variables to guess the values of the dependent variable. But in many real-world situations, classic regression models like simple and multiple linear regression generally presume that the connection between variables stays the same over the whole dataset. At some moments called breakpoints, the connection between two things changes. This is why regression models were created: they allow for a more realistic description of nonlinear interactions. Regression models are essential for examining data with structural changes; yet, pinpointing breakpoints and estimating model parameters pose significant obstacles, particularly when the data is compromised by noise or imprecision. In this situation, fuzzy logic has become a useful mathematical tool for dealing with uncertainty and ambiguity. It lets you show data using degrees of belonging instead of exact quantities, which makes it easier to simulate real-world events. Guresen and Kayakutlu (2011) created the theoretical groundwork for fuzzy logic. Since then, this discipline has become a key part of many scientific and technical fields. Fuzzy logic is useful for processing systems that are hard to correctly describe using standard models because it can mathematically express human knowledge. Artificial neural networks, on the other hand, have come a long way and are now one of the most essential tools in AI. They are unique because they can learn from data and find complicated patterns without needing to make any assumptions about how the variables are related. These networks have shown that they are quite good at many things, such recognizing patterns, making predictions, and processing signals. Adaptive Neuro-Fuzzy Inference Systems (ANFIS) are sophisticated hybrid models that use the best parts of both fuzzy logic and neural networks. These systems use fuzzy logic to explain things and neural networks to learn, which makes them great for developing models that are both accurate and adaptable. People often utilize ANFIS systems to figure out the parameters of nonlinear models, particularly when the data isn't very accurate or has some unknowns. They also make it possible to automatically find fuzzy rules in the data,

which makes predictions more accurate and cuts down on mistakes. When using fuzzy regression models, ANFIS may be used to find refraction points and estimate model parameters more quickly than other approaches. Numerous studies and research papers have addressed the topic of breakpoint regression and adaptive neural fuzzy inference systems. [Azizi et al. \(2015\)](#) aimed to model production uncertainty using an adaptive fuzzy neurology inference system (ANFIS) and compared it with a multiple linear regression model. They concluded that the ANFIS model achieved higher prediction accuracy. [Hamaamin et al. \(2016\)](#) used Bayesian regression models and adaptive fuzzy neurology inference systems (ANFIS) to estimate streamflow. They found that ANFIS models demonstrated a high capacity for representing nonlinear relationships and handling data uncertainty compared to traditional Bayesian models. [Sadeghifar and Barati \(2018\)](#) used ANFIS to estimate the longshore sediment transport rate on the southern coasts of the sea. They concluded that the ANFIS model had higher estimation accuracy compared to traditional methods. [Souza et al. \(2019\)](#) proposed a Bayesian fuzzy clustering-based neural network model for estimating regression models and concluded that the proposed model has a high capacity for representing both linear and nonlinear relationships. [Laaboudi and Slama \(2020\)](#) used fuzzy neural and linear models to estimate reference evapotranspiration in the southern region of Algeria. The study concluded that the ANFIS model achieved higher prediction accuracy compared to linear models, due to its ability to represent nonlinear relationships and handle complexity and uncertainty in environmental data, thus confirming its efficiency in real-world applications. [Ciuperca and Dulac \(2022\)](#) estimated the points of change in multivariate regression models that contain structural changes. They concluded that the proposed methodology provides accurate and efficient estimates of the positions of change points, even in complex multivariable models, thus contributing to improved accuracy in modeling and understanding the structural behavior of data. In the same year, [Al-Azzeh et al. \(2022\)](#) applied the Segmented Regression method to divide the data into homogeneous parts, and they concluded that the use of segmented regression contributes to increased accuracy of estimation and prediction compared to traditional models.

[Ruya and Gunal \(2023\)](#) compared three different models: Adaptive Fuzzy Neural Inference System (ANFIS), Gaussian Process Regression (GPR), and Simple Membership Fuzzy Rule Generation Technique (SMRGT). They concluded that the SMRGT model achieved the best performance in terms of estimation accuracy compared to the other models, while the ANFIS model showed good efficiency in dealing with nonlinear relationships. This confirms the effectiveness of intelligent models in regression and estimation problems. In the same year, [Oladipo and Sun \(2023\)](#) proposed an improved Adaptive Fuzzy Neural Inference System (ANFIS) model using the Genetic Algorithm (GA) to predict electrical energy consumption and improve the model's predictive efficiency. They found that the improved model achieved higher predictive accuracy compared to the traditional ANFIS model and demonstrated effective representation of nonlinear relationships and handling of complex data. [Kim et al. \(2024\)](#) developed a process for identifying refraction points in piecewise regression models by proposing new algorithms based on the Greedy Algorithm. They found that the proposed method is characterized by rapid convergence and stability in finding optimal refraction point locations. [Sagias et al. \(2024\)](#) used the Adaptive Fuzzy Neural Inference System (ANFIS) for predictive modeling of mechanical properties in additive manufacturing techniques. They found that the proposed model has high predictive accuracy and an effective ability to represent complex relationships between variables, making it a suitable tool in engineering applications that require accurate modeling under non-ideal data. [Pakdel et al. \(2025\)](#) estimated the coefficients of a fuzzy regression model using both the Adaptive Fuzzy Neural Inference System (ANFIS) and Bayesian methods. They found that combining ANFIS with Bayesian methods provides more accurate and efficient estimates of fuzzy regression model parameters. In the same year, [Thamaraimanalan et al. \(2025\)](#) used an adaptive fuzzy neural inference system (ANFIS) to analyze cognitive patterns based on multi-mode brain signals to process complex, nonlinear brain data, aiming to extract and analyze cognitive patterns accurately. They found that the proposed model has a high capacity for representing nonlinear relationships and handling multi-source data, and demonstrated efficiency in detecting hidden patterns within the data. Also, [Chen et al. \(2025\)](#) proposed a dynamic incremental adaptive fuzzy model based on the Takagi-Sugeno-Kang fuzzy classifier. They found that the proposed model is characterized by high classification efficiency and better ability to adapt to dynamic changes in the data.

2. Segmented Regression

Also known as broken-stick regression, this is a regression technique used when the relationship between the independent variable X and the dependent variable Y is not a single line across the entire domain, but rather changes at one or more points. It is a type of linear regression model, where the relationship between the response variable and one or more explanatory variables is assumed to be linear within multiple segments, each represented by an independent straight line. These segments are separated by points known as segments, which represent the values at which the behavior of the linear relationship changes. These segments are unknown and are determined based on specific statistical criteria or estimated directly from the data using specialized algorithms ([Muggeo, 2008](#)). The segmented regression model is characterized by the continuous nature of the relationship between the variables across the entire data domain, where no discontinuity occurs at the points of change. The adjustment merely affects the slope of the connection between the distinct

parts. So, the predicted values of the function stay continuous and don't hop about, which is a better way to show how the phenomena being studied really works. This model's arithmetic uses what are called Hinge functions to show how the slope changes after specific points. Due to these characteristics, segmented regression has found wide application in various fields, particularly in medicine, epidemiology, toxicology, occupational medicine, and environmental science. It is also commonly used in Interrupted Time Series (ITS) analysis to assess the impact of interventions and policies over time (Wang et al., 2025).

Assuming that the relationship between the response variable and the explanatory variable(s) does not follow a single linear pattern across the entire dataset, but rather can be represented as multiple linear segments separated by unknown points of change known as inflection points (τ), this type of model is expressed by a general function that considers the regression coefficients of each segment, as well as the positions of the inflection points that define the boundaries of these segments. Therefore, a breakpoint regression model can be formulated mathematically as follows: (Roslyakova, 2017; Wang et al., 2025).

$$y_i = f(x_i, \beta, \tau) + \varepsilon_i \quad ; i = 1, 2, \dots, n \quad (1)$$

y_i Dependent (response) variable of observation i , x_i is the explanatory (independent) variable of observation i . It may be a single variable or a vector of variables. $f(x_i, \beta, \tau)$ The segmented regression function that describes the relationship between the dependent variable and the explanatory variables. It takes the form of a cross-sectional function whose characteristics change at the inflection points. β regression coefficient vector, which includes the slope (and constant) coefficients for each cross-section of the model. τ the inflection point vector (or change points), which are the values at which the slope or shape of the relationship between the variable's changes. These are often unknown and are estimated from the data. ε_i Random error limits of observation i , representing the influence of factors not shown in the model. It is usually assumed to follow a normal distribution with a zero mean and a constant variance, i.e., $\varepsilon_i \sim N(0, \sigma^2_1)$. A segmented regression function can be represented as a segmental function consisting of several sub functions, where each segment of the independent variable's domain is assigned an independent linear regression model with its own coefficients. The boundaries of these segments are defined by the inflection points $\tau_1, \tau_2, \dots, \tau_{r-1}$, which divide the independent variable's domain into r consecutive intervals. Therefore, the regression function can be expressed as in Equation (2), where each sub function $f(x_i, \beta, \tau)$ represents the linear relationship within its corresponding interval.

$$f(x_i, \beta, \tau) = \begin{cases} f_1(x: \beta) & x_1 \leq x \leq \tau_1 \\ f_2(x: \beta) & \tau_1 \leq x \leq \tau_2 \\ \vdots & \\ f_r(x: \beta) & \tau_{r-1} < x \leq \tau_r \end{cases} \quad (2)$$

Where $f(x_i, \beta, \tau)$ are different subfunctions within different intervals $[\tau_{j-1}, \tau_j]$, and the endpoints of these intervals are unknown and must be estimated, these endpoints connect each subfunction to the next. These endpoints are usually important because they represent shifts or changes in the pattern and direction of the data. Therefore, partition regression can be called multistage or multiphase regression. β_j Regression coefficient vector for segment j , $\tau_1, \tau_2, \dots, \tau_{r-1}$ are the breakpoints (points of change) that divide the x -intercept, and r is the number of segments in the model. Assuming that the partition regression model is constrained by the condition of continuity at the breakpoints (points of change), the expected values of the function are equal at these points; that is, there is no discontinuity in the function, but rather the change is limited to the slope of the relationship between the different segments. Therefore, the breakpoint regression function can be represented as sub-linear functions $f(x_i, \beta, \tau)$, where each segment is assigned a linear model with specific coefficients, and the boundaries of these segments are defined by the inflection points $\tau_1, \tau_2, \dots, \tau_{r-1}$.

Under this constraint, the model can be defined as follows:

$$f(x_i, \beta, \tau) = \begin{cases} f_1(x: \beta) = \beta_{1,0} + \beta_{1,1}x & x_1 \leq x \leq \tau_1 \\ f_2(x: \beta) = \beta_{2,0} + \beta_{2,1}x & \tau_1 \leq x \leq \tau_2 \\ \vdots & \\ f_r(x: \beta) = \beta_{r,0} + \beta_{r,1}x & \tau_{r-1} < x \leq \tau_r \end{cases} \quad (3)$$

Assuming that the segmented regression model is constrained by the continuity condition at the points of change, the expected values of the function are equal at these points; that is, the end of each line segment coincides with the beginning of the next, thus ensuring no discontinuity in the function. This constraint is expressed mathematically as follows:

$$f(x_i, \beta, \tau) = \begin{cases} \beta_{1,0} + \beta_{1,1}x_i & x_i \leq \tau_1 \\ \beta_{1,0} + \beta_{1,1}\tau_1 + \beta_{2,0}(x_i - \tau_1) & \tau_1 \leq x \leq \tau_2 \\ \vdots & \\ \beta_{1,0} + \beta_{1,1}\tau_1 + \beta_{2,0}(x_i - \tau_1) + \dots + \beta_{r,1}(x_i - \tau_{r-1}) & x_i > \tau_{r-1} \end{cases} \quad (4)$$

By considering the continuity constraint at change points, the segmented regression model can be reformulated into a unified form known as the Hinge Function. In this form, the model is expressed using a single base slope, with cumulative adjustments to the slope after each change point. This formulation is more efficient in terms of the number of parameters and easier to estimate and interpret, as follows:

$$f(x_i, \beta, \tau) = \beta_0 + \beta_1 x + \Delta_1(x - \tau_1)_+ + \dots + \Delta_{r-1}(x - \tau_{r-1})_+ \quad (5)$$

Or it can be written in the following format: (Al-Azzeh et al., 2022)

$$f(x_i, \beta, \tau) = \beta_0 + \beta_1 x + \sum_{j=1}^{r-1} \Delta_j (x_i - \tau_j)_+ \quad (6)$$

Where β_0 is the parameter of the constant term (intercept), β_1 is the parameter of the slope in the first segment (before the first change point), Δ_j is the magnitude of the change in slope at the change point τ_j , and is defined as:

$$\Delta_j = \beta_{j+1} - \beta_j \quad (7)$$

$(x_i - \tau_j)_+$ positive part function called Hinge Function, and it is defined as follows:

$$(x_i - \tau_j)_+ = \begin{cases} x_i - \tau_j & x_i > \tau_j \\ 0 & x_i \leq \tau_j \end{cases} \quad (8)$$

τ_j Number j segments point.

Figure (1) shows the segmented regression model.

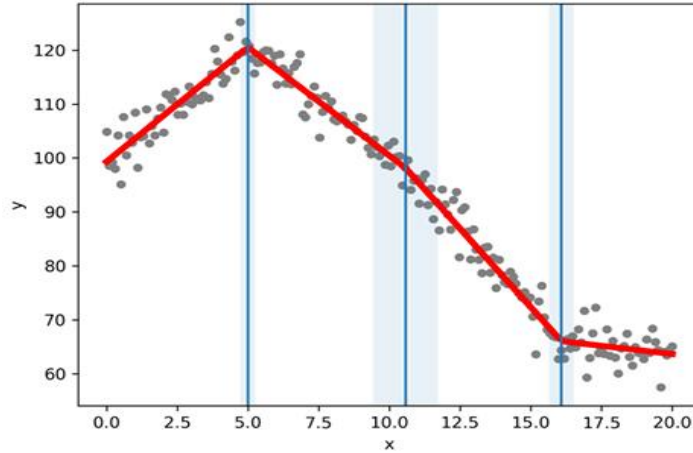


Figure 1: The Segmented Regression Model

3. Breakpoint Regression Model

Also known as regression at points of change, this is a statistical model used to represent relationships where the mathematical relationship between variables undergoes a genuine change, rather than simply random or natural fluctuations in data at one or more breakpoints within the data set of explanatory variables. This model allows for changes in the slope of the relationship, the constant term, or both at these points. It does not require continuity at the breakpoints, but rather allows for jumps in the expected values of the function. This makes it more flexible and general compared to the segmented regression model, which requires continuity. Consequently, it is suitable for analyzing phenomena that experience sudden changes or sharp shifts in their behavior. It can be argued that every segmented regression model is a breakpoint regression model, but not every breakpoint regression model is a segmented regression (Hamilton, 1994; Muggeo, 2008).

In the case the expected values of the function are not required to be equal at the points of regression; that is, the end of one segment does not have to coincide with the beginning of the next. Therefore, a discontinuity or a jump can occur at the point of regression. Thus, each segment can be represented by a linear function completely independent of the others, as follows:

$$f(x_i, \beta, \tau) = \begin{cases} \beta_{1,0} + \beta_{1,1}x_i & x_i \leq \tau_1 \\ \beta_{2,0} + \beta_{2,1}x_i & \tau_1 \leq x_i \leq \tau_2 \\ \vdots & \vdots \\ \beta_{r,0} + \beta_{r,1}x_i & x_i > \tau_{r-1} \end{cases} \quad (9)$$

Equation (9) demonstrates that transitioning from one segment to another may result in a change in the slope, the constant term, or both. Therefore, the breakpoint regression model is more general and flexible than the segmental regression model. Furthermore, the breakpoint regression model can be rewritten in a unified form using functional variables and components of the positive breakpoints that allowing for variations in the overall plane of the function at the points of inflection, as follows:

$$f(x_i, \beta, \tau) = \beta_0 + \beta_1 x + \gamma_1 I(x_i > \tau_1) + \dots + \gamma_{r-1} I(x_i > \tau_{r-1}) + \Delta_1 (x_i - \tau_1)_+ + \dots + \Delta_{r-1} (x_i - \tau_{r-1})_+ \quad (10)$$

Equation (10) can be r-written as follows:

$$f(x_i, \beta, \tau) = \beta_0 + \beta_1 x + \sum_{j=1}^{r-1} \gamma_j I(x_i > \tau_j) + \sum_{j=1}^{r-1} \Delta_j (x_i - \tau_j)_+ \quad (11)$$

Where γ_j jump at the point of refraction, τ_j , $I(x_i > \tau_j)$ index function, and is defined as follows: (Bai & Perron, 2002; Seber & Wild, 2003).

$$I(x_i > \tau_j) = \begin{cases} 1 & x_i > \tau_j \\ 0 & x_i \leq \tau_j \end{cases} \quad (12)$$

Equation (16) shows that the breakpoint regression model allows for two types of change at each point of breakpoint:

1. A change in the overall plane of the function, represented by γ_j
2. A change in the slope, represented by Δ_j

This model is suitable for analyzing phenomena that undergo sudden shifts or abrupt transitions in behavior, as it is not limited to gradual changes in slope, as is the case in continuous breakpoint regression.

Figure (2) shows the breakpoint regression model.

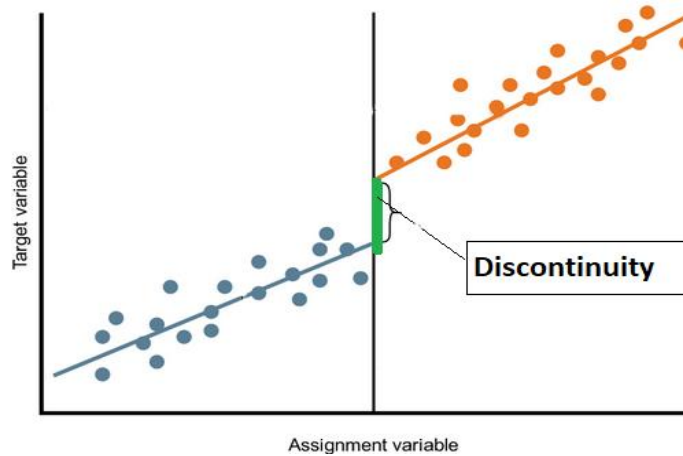


Figure 2: Breakpoint Regression Model

4. Traditional and Fuzzy Sets

A fuzzy set is defined as a set with non-sharp boundaries, where membership is not binary as in traditional sets. This concept allows for partial membership, providing a more realistic representation of phenomena characterized by ambiguity and imprecision.

Consider a sample space denoted by Ω , and a subset of it denoted by $A = \{x_1, x_2, \dots, x_n\}$, where $i = 1, 2, \dots, n$, each element x_i in A either belongs to or does not belong to the set A . Let $c_A(x)$ be a characteristic function of the set A , assigning a degree of membership to each element in the subset Ω , with a range of $\{0, 1\}$ such that:

$$c_A(x) = \begin{cases} 1, & \text{if } x \in A \\ 0, & \text{if } x \notin A \end{cases} \quad (13)$$

If $c_A(x) = 1$, then the element x_i belongs entirely to set A . If $c_A(x) = 0$, then the element x_i does not belong to set A at all.

Set A is called a crisp set (Saatchi, 2024). Figure (3) shows the crisp set where the domain is divided into three categories: Low, Medium, and High. These sets are characterized by sharp boundaries between them, as the membership function takes only binary values (0 or 1). This means that an element either belongs entirely to a particular set or does

not belong to it at all, with any intermediate degrees. This representation reflects the deterministic nature of traditional groups, which cannot express the ambiguity or gradation present in many real-world phenomena, unlike fuzzy groups that allow for partial belonging.

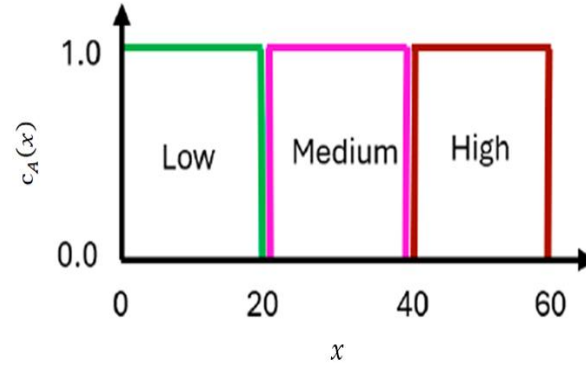


Figure 3: Traditional Crisp Set (Saatchi, 2024)

If we have a subset denoted $\tilde{A}$ of Ω that is characterized by a membership function denoted $\mu_{\tilde{A}}(x)$ that generates memberships whose limits are within the interval $[0,1]$ for all x_i values in the fuzzy sample space defined as follows:

$$\tilde{A} = \{(x_i, \mu_{\tilde{A}}(x_i)), x \in \Omega, i = 1, 2, 3, \dots, n, 0 \leq \mu_{\tilde{A}}(x) \leq 1\} \quad (14)$$

$\tilde{A}$ set that allows partial belonging, each element in it has a certain degree of belonging. Figure (4) shows the fuzzy set, where we observe the division of the domain into three language categories: Low, Medium, and High. Unlike traditional sets, these sets are characterized by overlap, allowing a single element to belong to more than one set to varying degrees. The values on the vertical axis represent the degree of membership ranging from 0 to 1. The dashed lines represent an example of a specific value of x having different degrees of membership in more than one set simultaneously.

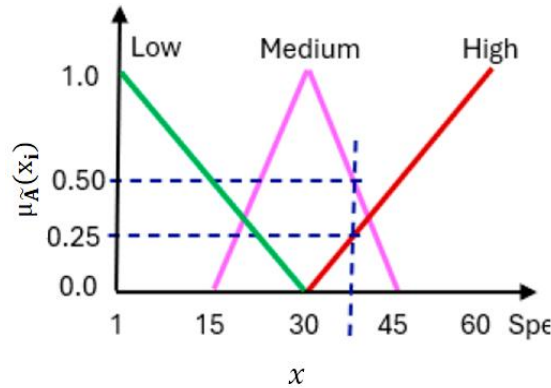


Figure 4: Fuzzy Set (Saatchi, 2024)

5. Membership Functions

It is the function that assigns membership degrees to each value in the sample space within the interval $[0, 1]$. These values express the degree of membership of each element, rather than the binary membership found in traditional sets (Saatchi, 2024). The membership function can be viewed as a positive function describing the importance of an element or its degree of membership in the fuzzy set. It represents a generalization of the characteristic function in traditional sets, where values between 0 and 1 allow for the representation of partial membership degrees, reflecting the inherent ambiguity and imprecision of the data (Pedrycz & Vukovich, 2002).

Membership functions are characterized by three main properties:

1. Core: If $\tilde{A}$ is a fuzzy set with a membership degree of 1.
2. Support: If $\tilde{A}$ is a fuzzy set, then the elements of $\tilde{A}$ have a membership degree greater than zero, as follows:

$$\text{Support}(\tilde{A}) = \{x \in \Omega / \mu_{\tilde{A}}(x) > 0\} \quad (15)$$

- 3- Boundaries: If $\tilde{A}$ is a fuzzy set, then the elements of $\tilde{A}$ that have a degree of membership greater than zero and are

not perfect belonging as follows:

$$\text{Boundary } (\tilde{A}) = \{x \in \Omega; 0 < \mu_{\tilde{A}}(x) < 1\} \quad (16)$$

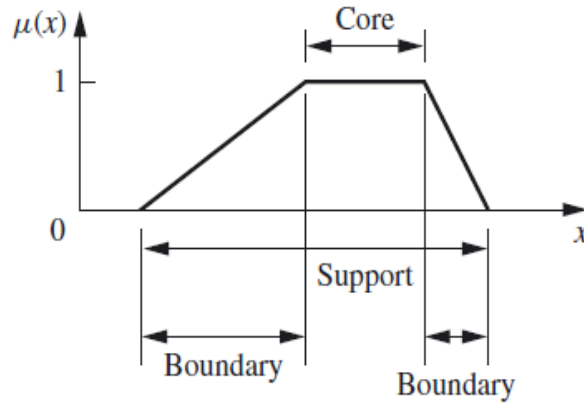


Figure 5: Features of the Membership Function (Ross, 2010)

6. Adaptive Neuro-Fuzzy Inference System

Following the emergence of fuzzy logic theory, researchers [Takagi and Sugeno \(1985\)](#) proposed a fuzzy inference system model based on adaptive networks, known as ANFIS to represent the relationship between inputs and outputs using fuzzy rules and appropriate membership functions. This system relies on generating input-output pairs using a set of fuzzy rules constructed based on membership functions. To illustrate the model, we assume that the system has two inputs, X and Y , and one output, Z . We also assume that the system's rules consist of two Takagi-Sugeno fuzzy rules, as follows:

Rule 1:

If x is A_1 and y is B_1 , then $f_1 = p_1x + q_1y + r_1$,

Rule 2:

If x is A_2 and y is B_2 , then $f_2 = p_2x + q_2y + r_2$

ANFIS consists of five layers:

Layer (1): This represents the membership function layer. It inputs the variables X and Y . Each variable passes through a fuzzy membership function $\mu_{A_i}(x) = O_i^1$

x input to node i , A_i linguistic classification is the description we use to express the value of the variable instead of using a specific number, such as small or large. O_i membership function for A_i that determines the degree to which x satisfies the given quantum value of A_i . We usually choose $\mu_{A_i}(x)$ for general Gaussian, as follows:

$$\mu_{A_i}(x) = \frac{1}{1 + \left(\frac{x-c}{a}\right)^2)^b} \quad (17)$$

The values a , b , and c are fundamental parameters that control the shape of the membership function. As the values of these parameters change, the bell-shaped functions change accordingly, exhibiting different forms of membership functions on the linguistic description A_i .

Layer (2): Finding the weight of each base. Each node in this layer is a circular node labeled Π that multiplies the functions.

$$w_i = \mu_{A_i}(x) \cdot \mu_{B_i}(y) \quad (18)$$

Layer (3): In this layer, the nodes are in the form of circles and are designated N . The node applies the normalization process so that the activation forces of each base are relative to the total activation forces in the system.

$$\bar{w}_i = \frac{w_i}{w_1 + w_2} \quad i = 1, 2, \dots \quad (19)$$

Layer (4): In the fourth layer, each node, denoted by the symbol i , is a square-shaped node that executes its own function.

$$O_i^4 = \bar{w}_i (p_1x + q_1y + r_1) \quad (20)$$

w_i output of the third layer. p , q and r a set of parameters. The parameters in this layer are referred to as dependent

parameters because they are directly related to the outputs of the fuzzy rules.

Layer (5): The node in this layer is circular in shape. The function of this node is to collect the signals coming to it from the previous layers and calculate the total output of the system.

$$O_1^5 = \text{overall output} = \sum \bar{w}_i f_i = \frac{\sum w_i f_i}{w_i} \quad (21)$$

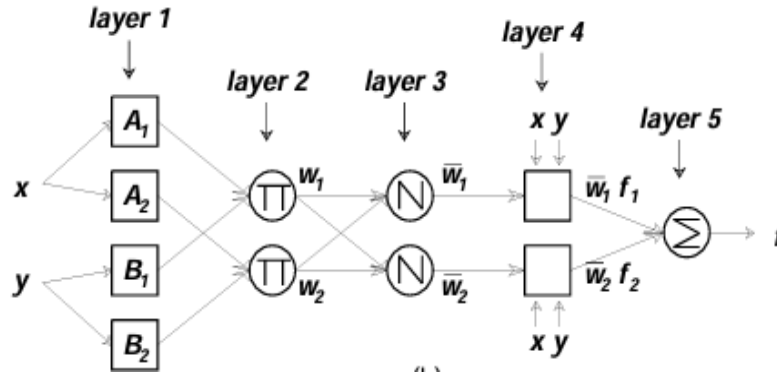


Figure 6: ANFIS Layers

7. Real Data Set

A set of health variables was used to assess the risk of cardiovascular complications. These variables were age (Age_years), body mass index (BMI), blood glucose level (Glucose_mg_dL), systolic blood pressure (SBP_mmHg), cholesterol level (Cholesterol_mg_dL), and C-reactive protein (CRP_mg_L). The data were obtained from <https://data.mendeley.com/> using 200 observations. Two models, the breakpoint and regression models were estimated using this data. First, the ordinary least squares method was used, and then the adaptive neural fuzzy inference model was used. The data were then tested and fitted for analysis using the CUSUM test and the Chow test. The results were as follows:

Table 1: CUSUM Test Results for Model Stability Over Time

Recursive Residuals No.	Sigma Recursive Residuals	Min CUSUM	Max CUSUM
193	5.06	-0.056	59.64

Table (1) shows that the model parameters are not stable over time. The path of the residuals is of a gradual rise and therefore does not support the hypothesis of stability. Consequently, there is a variation in the relationship between the variables, meaning that the trends differ over time and are not constant.

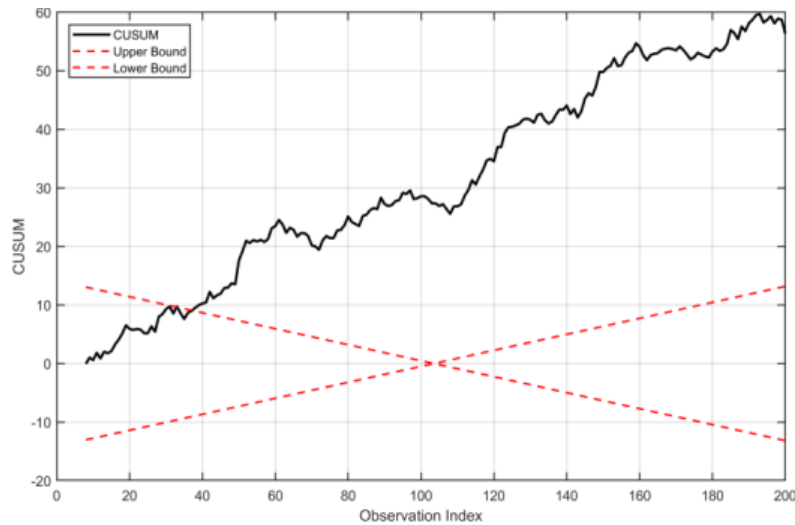


Figure 7: CUSUM Test Cumulative Curve

Figure (7) shows that the cumulative curve of the residuals adjusts the confidence limits and that the parameters are:

Table 2: Chow Test Results

Breakpoint	Index	N _{Before}	N _{After}	SSE _{Pooled}	SSE _{Breakpoint}	F _{Statistic}	P-Value
110	43	43	157	4939.06	4359.11	3.54	0.000
160	108	108	92	4939.06	4392.95	3.30	0.000

The results in Table (2) reveal a structural change in the data at two potential points, 110 and 160, according to Chow's test. This rejects the null hypothesis, which assumes the absence of structural change, indicating that the relationship between variables before this point differs significantly from that after. Similarly, at the refracting point 160, the P-value is (0.00245), also less than 0.05, confirming the presence of a second structural change at this stage. Furthermore, the lower value of SSE_{Breakpoint} compared to SSE_{Pooled} in both cases suggests that breakpoints the data into two periods improves model fit, which supports the existence of this change.

Therefore, the segmented regression and breakpoint regression models can be estimated as follows:

First: Estimation using the ordinary least squares method:

Table 3: Comparison of the Split Regression Model and the Regression Model Using OLS

Model	Breakpoints No.	MSE	RMSE	R ²	AIC	BIC
Segmented Regression	2	24.35	4.82	0.90	647.29	676.98
Breakpoint Regression	2	22.35	4.60	0.91	632.01	668.29

Table (3) shows that the Breakpoint Regression model slightly outperforms the Segmented Regression model at the same breakpoints, with higher MSE and RMSE values and a higher coefficient of determination (R²), indicating a better explanation of the variance in the data. Furthermore, the lower AIC and BIC values in the breakpoint regression model suggest a better fit to the model.

Table 4: Summary of Residual Statistics for Segmented Regression and Breakpoint Regression Models Using OLS Method

Model	Mean Residual	Std. Residual	Min Residual	Max Residual
Segmented Regression	0.00	4.83	-12.88	16.38
Breakpoint Regression	0.00	4.61	-12.07	13.40

Table (4) shows that both models have a mean residual of 0.00, indicating no estimation bias. However, the breakpoint regression model performs better in terms of error dispersion, recording a lower standard deviation of 4.61 compared to 4.83 in the split regression model, suggesting a greater convergence of the residuals around zero.

The estimation equation for the segmented regression model using OLS is as follows:

$$\begin{aligned}\hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i3} + \hat{\beta}_4 x_{i4} + \hat{\beta}_5 x_{i5} + \hat{\beta}_6 x_{i6} + \hat{\Delta}_1 (x_i - \tau_1)_+ + \hat{\Delta}_2 (x_i - \tau_2)_+ \\ \widehat{ComplicationRiskScore}_i &= \hat{\beta}_0 + \hat{\beta}_1 Glucose_i + \hat{\beta}_2 Age_i + \hat{\beta}_3 BMI_i \\ &\quad + \hat{\beta}_4 SBP_i + \hat{\beta}_5 Cholesterol_i + \hat{\beta}_6 CRP_i + \hat{\Delta}_1 (Glucose_i - 110)_+ + \hat{\Delta}_2 (Glucose_i - 160)_+ \\ \widehat{ComplicationRiskScore}_i &= -5.99 + 0.13Glucose_i + 0.03Age_i + 0.17BMI_i + 0.06SBP_i + 0.01Cholesterol_i + \\ &\quad 0.75CRP_i + 0.17(Glucose_i - 110)_+ - 0.005(Glucose_i - 160)_+ \quad (22)\end{aligned}$$

The estimated equation for breakpoint regression model using OLS is as follows:

$$\begin{aligned}\hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i3} + \hat{\beta}_4 x_{i4} + \hat{\beta}_5 x_{i5} + \hat{\beta}_6 x_{i6} + \gamma_1 I(x_i > \tau_1) + \gamma_2 I(x_i > \tau_2) + \hat{\Delta}_1 (x_i - \tau_1)_+ \\ &\quad + \hat{\Delta}_2 (x_i - \tau_2)_+ \\ \widehat{ComplicationRiskScore}_i &= 3.87 + 0.01Glucose_i + 0.03Age_i + 0.20BMI_i + 0.06SBP_i + 0.02Cholesterol_i + 0.75CRP_i + \\ &\quad 6.05I(Glucose_i > 110) + 5.83I(Glucose_i > 160) + 0.16(Glucose_i - 110)_+ + 0.10(Glucose_i - 160)_+ \quad (23)\end{aligned}$$

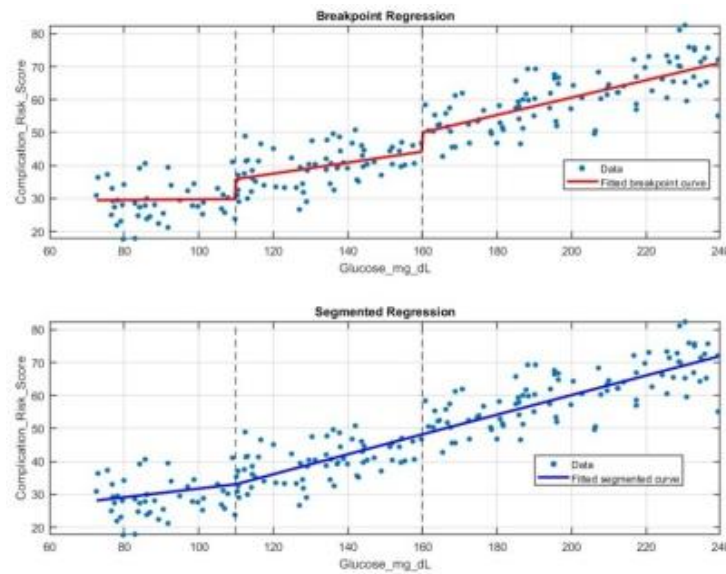
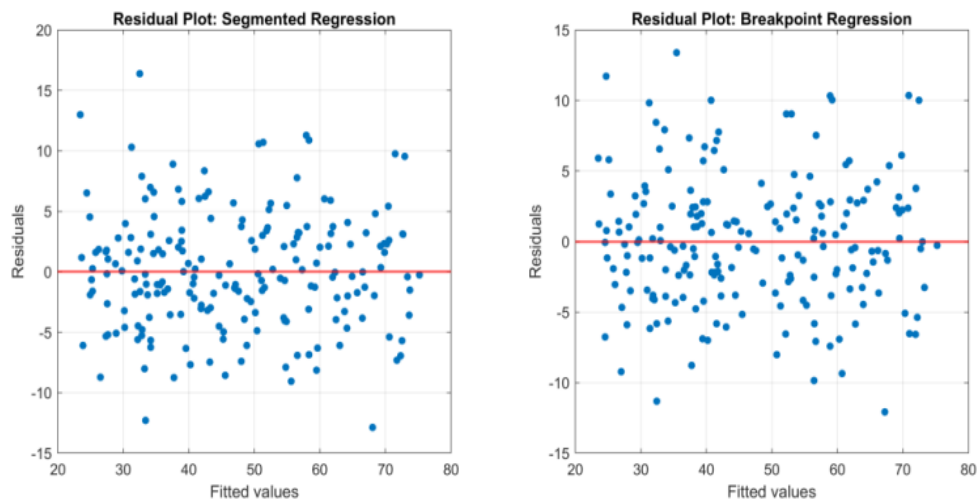
Table 5: (a) Actual, Estimated Values of the Dependent Variable with Residuals and Error Parameters for the Segmented and Breakpoint Regression Models using OLS

i	Glucose	y	Segmented Model				Breakpoint Model			
			$\hat{y}$	Residual	MSE	RMSE	$\hat{y}$	Residual	MSE	RMSE
1	72.6	30.9	24.4	6.5	42.4	17.9	25.1	5.8	33.8	5.8
2	72.9	36.4	23.4	13.0	169.0	10.4	24.7	11.7	137.4	11.7
3	75.5	37.3	34.2	3.1	9.6	31.1	34.8	2.5	6.2	2.5
4	76.6	24.9	27.6	-2.6	7.0	30.2	28.4	-3.5	12.1	3.5
5	76.7	33.1	32.1	0.9	0.8	31.2	33.0	0.1	0.0	0.1
6	77.0	29.2	27.5	1.8	3.1	25.7	28.2	1.0	1.1	1.0

Table 5: (b) Actual, Estimated Values of the Dependent Variable with Residuals and Error Parameters for the Segmented and Breakpoint Regression Models using OLS

i	Glucose	y	Segmented Model					Breakpoint Model		
			$\hat{y}$	Residual				$\hat{y}$	Residual	
7	77.4	27.4	25.8	1.6	2.5	24.2	26.7	0.7	0.5	0.7
8	78.1	22.0	27.3	-5.3	28.3	32.6	27.9	-5.9	34.8	5.9
9	78.2	29.4	28.8	0.6	0.4	28.1	29.5	-0.1	0.0	0.1
10	79.1	23.0	24.9	-1.9	3.7	26.8	26.1	-3.0	9.2	3.0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
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Table (5) and Figure (8) show that both the segmented and breakpoint regression models are capable of approximating the true values of the dependent variable. However, the breakpoint regression model exhibits greater accuracy in most observations, with estimated values being closer to the true values and residuals are lower in most cases. This is reflected in the lower MSE and RMSE values compared to the fractional model. Furthermore, the differences between the true and estimated values in the fractional model are less volatile and more stable, indicating a better ability to represent the relationship between the variables. This confirms the superiority of the breakpoint regression model in terms of accuracy and predictive efficiency.

**Figure 8:** Curve of the Segmented Regression Model and the Breakpoint Regression at Two Points of Refraction, Estimated using OLS**Figure 9:** Residues of the Segmented Regression Model and the Breakpoint Regression at Two Breakpoints Estimated using OLS

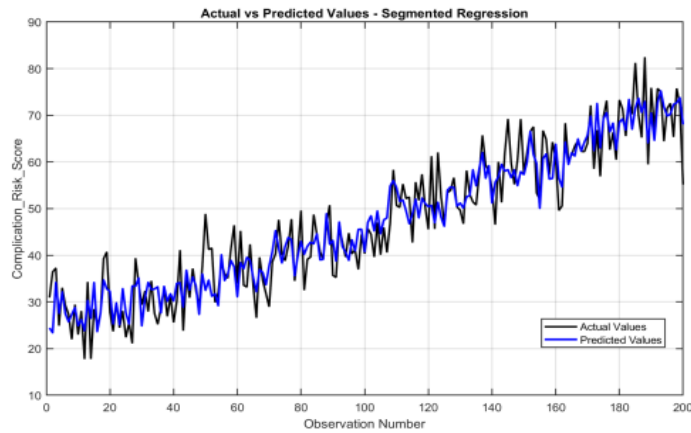


Figure 10: Actual and Estimated Values of the Dependent Variable for the Segmented Regression Model Estimated using OLS

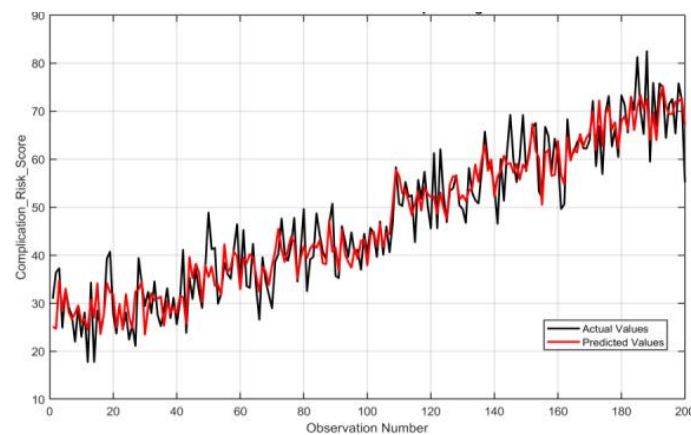


Figure 11: Actual and Estimated Values of the Dependent Variable for Breakpoint Regression Model Estimated using OLS

Second: Estimation using the Adaptive Neural Fuzzy Inference Model

Table 6: Comparison of the Segmented Regression Model and the Breakpoint Regression Model using ANFIS

Model	MSE	RMSE	R ²	AIC	BIC
ANFIS	9.45	4.55	0.81	1056.56	1076.45
ANFIS Segmented Regression	7.27	2.70	0.92	1022.21	1026.91
ANFIS Breakpoint Regression	2.12	1.46	0.94	1010.12	1011.22

Table (6) shows the superiority of the improved ANFIS models over the basic ANFIS model. The ANFIS breakpoint regression model performed best among all the models, recording the lowest MSE value of 2.12 and the lowest RMSE of 1.46, along with the highest coefficient of determination (R^2) of 0.94, demonstrating very high predictive accuracy and excellent ability to explain data variance. The segmented ANFIS model performed well and outperformed the basic ANFIS model.

Table 7: Residual for the ANFIS Segmented and ANFIS Breakpoint Regression Models

Model	Mean Residual	Std. Residual	Min Residual	Max Residual
Baseline ANFIS	0.00	2.35	-42.88	8.45
Segmented ANFIS	0.00	2.22	-127.07	9.14
Breakpoint ANFIS	0.00	1.89	-127.07	8.89

Table (7) demonstrates that all of the ANFIS models have a mean residual of 0.00, which means that there is no bias in the estimates. The Breakpoint ANFIS model, on the other hand, is the best at making predictions. It has the lowest standard deviation of residuals at 1.89, which means that mistakes are more likely to converge on zero and the model is more stable than the others. Next is the Segmented ANFIS model, which has a standard deviation of 2.22, and then the Baseline ANFIS model, which has a standard deviation of 2.35. Some models have very low minimum values, but the Breakpoint model has the lowest total residual dispersion. This shows that it is better at representing data and making more accurate predictions.

The estimated equation for the basic ANFIS model is as follows:

$$\begin{aligned}\hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i3} + \hat{\beta}_4 x_{i4} + \hat{\beta}_5 x_{i5} + \hat{\beta}_6 x_{i6} + \hat{\Delta}_1(x_i - \tau_1)_+ + \hat{\Delta}_2(x_i - \tau_2)_+ \\ &\quad \text{ComplicationRiskScore}_i \\ &= \hat{\beta}_0 + \hat{\beta}_1 \text{Glucose}_i + \hat{\beta}_2 \text{Age}_i + \hat{\beta}_3 \text{BMI}_i \\ &\quad + \hat{\beta}_4 \text{SBP}_i + \hat{\beta}_5 \text{Cholesterol}_i + \hat{\beta}_6 \text{CRP}_i + \hat{\Delta}_1(\text{Glucose}_i - 110)_+ + \hat{\Delta}_2(\text{Glucose}_i - 160)_+ \\ \text{ComplicationRiskScore}_i &= 1.89 + 2.23\text{Glucose}_i + 0.01\text{Age}_i + 0.11\text{BMI}_i + 0.01\text{SBP}_i + 0.05\text{Cholesterol}_i + \\ &\quad 0.34\text{CRP}_i + 0.35(\text{Glucose}_i - 110)_+ + 0.12(\text{Glucose}_i - 160)_+ \quad (24)\end{aligned}$$

The estimated equation for the ANFIS segmented regression model is as follows:

$$\begin{aligned}\hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i3} + \hat{\beta}_4 x_{i4} + \hat{\beta}_5 x_{i5} + \hat{\beta}_6 x_{i6} + \hat{\Delta}_1(x_i - \tau_1)_+ + \hat{\Delta}_2(x_i - \tau_2)_+ \\ &\quad \text{ComplicationRiskScore}_i \\ &= \hat{\beta}_0 + \hat{\beta}_1 \text{Glucose}_i + \hat{\beta}_2 \text{Age}_i + \hat{\beta}_3 \text{BMI}_i \\ &\quad + \hat{\beta}_4 \text{SBP}_i + \hat{\beta}_5 \text{Cholesterol}_i + \hat{\beta}_6 \text{CRP}_i + \hat{\Delta}_1(\text{Glucose}_i - 110)_+ + \hat{\Delta}_2(\text{Glucose}_i - 160)_+ \\ \text{ComplicationRiskScore}_i &= -7.23 + 0.33\text{Glucose}_i + 0.08\text{Age}_i + 0.15\text{BMI}_i + 0.09\text{SBP}_i + 0.02\text{Cholesterol}_i + \\ &\quad 0.85\text{CRP}_i + 0.85(\text{Glucose}_i - 110)_+ - 0.02(\text{Glucose}_i - 160)_+ \quad (25)\end{aligned}$$

The estimated equation for the breakpoint regression model using ANFIS is as follows:

$$\begin{aligned}\hat{y}_i &= \hat{\beta}_0 + \hat{\beta}_1 x_{i1} + \hat{\beta}_2 x_{i2} + \hat{\beta}_3 x_{i3} + \hat{\beta}_4 x_{i4} + \hat{\beta}_5 x_{i5} + \hat{\beta}_6 x_{i6} + \gamma_1 I(x_i > \tau_1) + \gamma_2 I(x_i > \tau_2) + \hat{\Delta}_1(x_i - \tau_1)_+ \\ &\quad + \hat{\Delta}_2(x_i - \tau_2)_+ \\ \text{ComplicationRiskScore}_i &= 5.44 + 0.56\text{Glucose}_i + 0.09\text{Age}_i + 0.22\text{BMI}_i + 0.09\text{SBP}_i + 0.05\text{Cholesterol}_i + \\ &\quad 0.98\text{CRP}_i + 9.12I(\text{Glucose}_i > 110) + 7.55I(\text{Glucose}_i > 160) + 0.23(\text{Glucose}_i - 110)_+ + 0.18(\text{Glucose}_i - 160)_+ \quad (26)\end{aligned}$$

Table 8: Actual and Estimated Values of the Dependent Variable with Residuals and Error Parameters for the Segmented Regression and Breakpoint Regression Models using ANFIS

i	Glucose	y	ANFIS				ANFIS Segmented Model				ANFIS Breakpoint Model			
			$\hat{y}$	Residual	MSE	RMSE	$\hat{y}$	Residual	MSE	RMSE	$\hat{y}$	Residual	MSE	RMSE
1	72.6	30.9	33.8	0.00	8.4	2.9	33.3	0.00	5.8	2.4	30.8	0.00	0.0	0.1
2	72.9	36.4	36.3	3.00	0.0	0.1	37.8	-3.00	2.0	1.4	36.3	0.00	0.0	0.1
3	75.5	37.3	35.2	5.00	4.4	2.1	35.7	-5.00	2.6	1.6	37.8	0.00	0.2	0.5
4	76.6	24.9	27.8	0.00	8.4	2.9	27.3	0.00	5.8	2.4	24.9	0.00	0.0	0.0
5	76.7	33.1	34.9	1.00	3.6	1.9	34.5	-1.00	2.0	1.4	33.7	0.00	0.5	0.7
6	77.0	29.2	30.1	2.00	0.8	0.9	30.6	-2.00	2.0	1.4	29.1	0.00	0.0	0.1
7	77.4	27.4	29.3	-1.00	3.6	1.9	29.2	1.00	3.2	1.8	29.1	0.00	2.9	1.7
8	78.1	22.0	23.9	0.00	3.6	1.9	23.4	0.00	2.0	1.4	23.2	0.00	1.4	1.2
9	78.2	29.4	26.3	0.00	9.6	3.1	28.8	0.00	0.4	0.6	28.9	0.00	0.2	0.5
10	79.1	23.0	25.9	0.00	8.4	2.9	25.4	0.00	5.8	2.4	24.4	0.00	2.0	1.4
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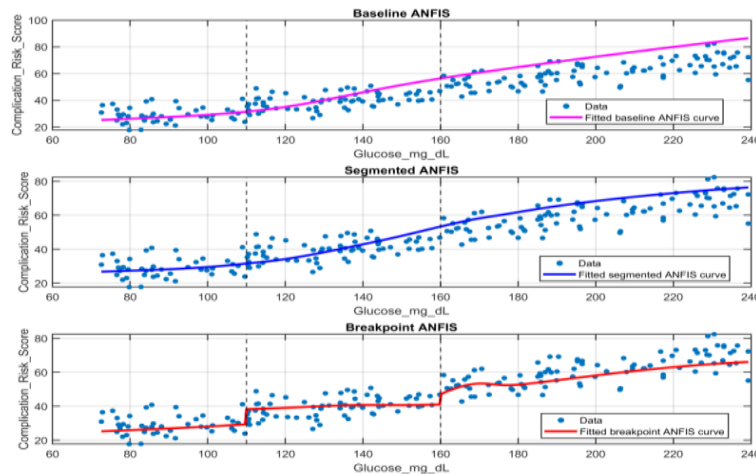


Figure 12: Shows the Curve of the Segmented Regression and Breakpoint Regression at Two Points of Refraction, Estimated using ANFIS

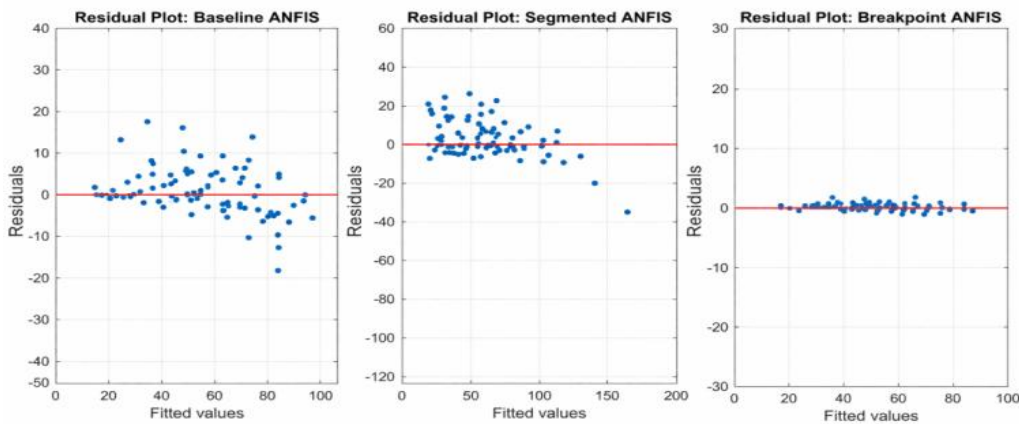


Figure 13: Residues of the Segmented and Breakpoint Regression Models at two Refraction Points Estimated using ANFIS'

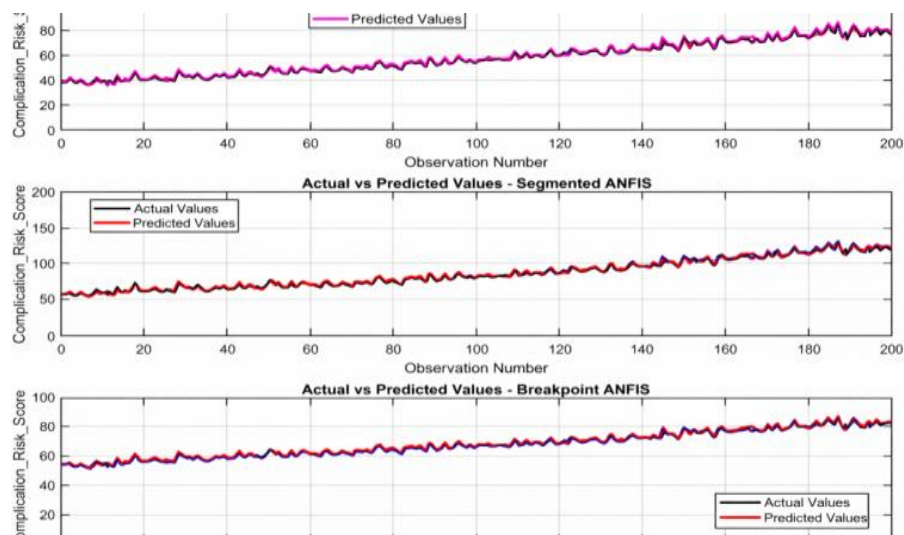


Figure 14: Actual and Estimated Values of the Dependent Variable for the Segmented, Breakpoint Regression Models Estimated using ANFIS

Based on the analysis results, it is clear that the ANFIS model estimates significantly outperform those of the Ordinary Least Squares (OLS) method. ANFIS models achieved a substantial reduction in error values such as MSE and RMSE compared to OLS models, decreasing from 22.35 and 4.60 in the best OLS model to 2.12 and 1.46 in the refractive ANFIS model. Furthermore, the coefficient of determination (R^2) increased from 0.91 to 0.94, indicating higher explanatory power. Additionally, the ANFIS residuals were less dispersed and more stable, with a high degree of convergence between the actual and estimated values, reflecting very high predictive accuracy. Therefore, it can be concluded that the ANFIS model, and especially the refractive model, is the best and most efficient in representing the data compared to OLS models.

Table 9: Comparative Analysis of the OLS ANFIS Model Coefficients and Their Statistical Significance for Health Variables

Variable	OLS Segmented	Sig	OLS Breakpoint	Sig	ANFIS	Sig	ANFIS Segmented	Sig	ANFIS Breakpoint	Sig
(Glucose)	0.13	0.01	0.01	0.08	2.23	0.000	0.33	0.001	0.56	0.000
(Age)	0.03	0.29	0.03	0.07	0.01	0.20	0.08	0.02	0.09	0.000
(BMI)	0.17	0.12	0.20	0.01	0.11	0.03	0.15	0.03	0.22	0.01
(SBP)	0.06	0.08	0.06	0.07	0.01	0.25	0.09	0.04	0.09	0.01
(Cholesterol)	0.01	0.20	0.02	0.15	0.05	0.07	0.02	0.04	0.05	0.01
(CRP)	0.75	0.13	0.75	0.000	0.34	0.001	0.85	0.001	0.98	0.000
(Glucose>110)	0.17	0.02	6.05+0.16	0.000	0.35	0.001	0.85	0.001	9.12+0.23	0.000
(Glucose>160)	-0.005	0.40	5.83+0.10	0.000	0.12	0.08	-0.02	0.02	7.55+0.18	0.001

Table (9) shows that the ordinary least squares segmented model yielded relatively weak results, as some important

variables, such as CRP and glucose, were considered less significant than expected at certain levels. This indicates its inability to capture the true complexity of the relationship, and thus it can be said that it shows some variables as insignificant despite their actual importance. The OLS segmented model significantly improved this picture, highlighting the significance of key variables such as CRP, glucose, and refraction points. However, it still showed relatively weak significance for some variables, such as age and cholesterol. Similarly, the basic ANFIS model showed improvement in highlighting the significance of key variables, but remained limited in explaining all relationships due to its general nature, which is not specific to refractions. The ANFIS breakpoint model provided better results, with most variables becoming reasonably significant, and with a clear improvement in estimating the impact of health variables.

8. Conclusions

The parameters of the model are not constant throughout time. The CUSUM test findings corroborated this by showing that the connection between the explanatory and dependent variables was always changing in terms of structure. This suggests that the nature of this link is not stable but is influenced by temporal variables or items excluded from the model. The Chow test findings supported this hypothesis by identifying two major refraction spots at glucose levels (110) and (160), indicating considerable changes in data behavior. When you compare regression models that were estimated using the ordinary least squares approach, the breakpoint regression model is better than the segmented regression model in terms of how well it fits the data and how accurately it estimates. This is shown by lower error values, stronger explanatory power, and better residual characteristics, which are less spread out and more stable. Linear models can get close to real values, but they can't fully capture complicated nonlinear interactions. The Adaptive Neural Fuzzy Inference Model (ANFIS), on the other hand, showed a clear advantage over OLS models. It had much lower error values, higher coefficients of determination, and a remarkable agreement between actual and estimated values, which shows that it was very accurate at making predictions. The breakpoint ANFIS model was the best of all the models since it could best represent data, had the lowest error rates, and had the most explanatory power. These findings stress the need of integrating refraction points into statistical models, particularly when addressing dynamic and nonlinear health data, and emphasize the vital function of hybrid models in improving the quality of analysis and prediction. It can also be said that the health factors that were evaluated, especially glucose and C-reactive protein levels, have a big impact on explaining the risk of problems. This shows how important it is to use sophisticated models that can correctly capture these impacts. Moreover, the statistical significance findings for the variables indicated that the models varied in their capacity to emphasize the relevance of factors. The segmented OLS model proved inadequate in identifying the significance of critical variables, including CRP and specific glucose effects, sometimes misrepresenting them as unimportant despite their true relevance, hence highlighting its shortcomings in capturing complicated interactions. The refractive OLS model shown significant improvement in emphasizing the importance of principal variables and refraction points; nevertheless, it still had relative deficiencies for some minor factors. The simple ANFIS model made it easier to figure out how important each variable was, but it didn't adequately show how the structure changed. The segmented ANFIS model, on the other hand, performed better for most variables when it came to stability of significance. Finally, the ANFIS breakpoint model proved that it was better than the others not only because it was more accurate at making predictions, but also because it made the true importance of the variables clear. This was especially true for glucose, CRP, and refractive effects, which showed that the model was better at showing nonlinear relationships and structural changes.

9. References

- Al-Azzeh, J., Mesleh, A., Zaliskyi, M., Odarchenko, R., & Kuzmin, V. (2022). A method of accuracy increment using segmented regression. *Algorithms*, 15(10), 378.
- Azizi, A., Bin Ali, A., & Ping, L. (2015). Modelling production uncertainties using the adaptive neuro-fuzzy inference system. *South African Journal of Industrial Engineering*, 26(1), 224-234.
- Bai, J., & Perron, P. (2002). Computation and analysis of multiple structural change models. *Journal of Applied Econometrics*, 18(1), 1-22.
- Chen, H., Sha, C., Jiao, M., Shao, C., Gao, S., Yu, H., & Qin, B. (2025). DIA-TSK: A Dynamic Incremental Adaptive Takagi-Sugeno-Kang Fuzzy Classifier. *Mathematics*, 13(7), 1054.
- Ciuperca, G., & Dulac, N. (2022). Multiple Change-Points Estimation in Linear Regression Models via an Adaptive LASSO Expectile Loss Function. *Journal of Statistical Theory and Practice*, 16(3), 38.
- Guresen, E., & Kayakutlu, G. (2011). Definition of artificial neural networks with comparison to other networks. *Procedia Computer Science*, 3, 426-433.

- Hamaamin, Y. A., Nejadhashemi, A. P., Zhang, Z., Giri, S., & Woznicki, S. A. (2016). Bayesian regression and neuro-fuzzy methods reliability assessment for estimating streamflow. *Water*, 8(7), 287.
- Hamilton, J. D. (1994). Time Series Analysis. In: Princeton University Press.
- Kim, T., Lee, H., Kim, M., & Choi, H. (2024). Improved identification of breakpoints in piecewise regression and its applications. *arXiv preprint arXiv:2408.13751*.
- Laaboudi, A., & Slama, A. (2020). Using Neuro-fuzzy and linear models to estimate reference Evapotranspiration in South region of Algeria (A comparative study). *Italian Journal of Agrometeorology*(2), 55-64.
- Muggeo, V. M. (2008). Segmented: an R package to fit regression models with broken-line relationships. *R news*, 8(1), 20-25.
- Oladipo, S., & Sun, Y. (2023). Enhanced adaptive neuro-fuzzy inference system using genetic algorithm: A case study in predicting electricity consumption. *SN Applied Sciences*, 5(7), 186.
- Pakdel, M., Razzaghnia, T., Fathi, K., & Mostafaei, A. (2025). Estimation of Fuzzy Regression Parameters With ANFIS and Bayesian Methods. *Engineering Reports*, 7(1), e13086.
- Pedrycz, W., & Vukovich, G. (2002). On Elicitation of Membership Functions. *IEEE Transactions on Systems, Man, and Cybernetics. Part A: Systems and Humans*, 32(6), 761-767.
- Roslyakova, I. (2017). *Modeling thermodynamical properties by segmented non-linear regression* Ruhr University Bochum, Germany].
- Ross, T. J. (2010). Fuzzy Logic with Engineering Applications. (No Title).
- Ruya, M., & Gunal, A. Y. (2023). A comparative study of using adaptive neural fuzzy inference system (ANFIS), Gaussian process regression (GPR), and SMRGT models in flow coefficient estimation. *3c Tecnología: glosas de innovación aplicadas a la pyme*, 12(2), 125-146.
- Saatchi, R. (2024). Fuzzy logic concepts, developments and implementation. *Information*, 15(10), 656.
- Sadeghifar, T., & Barati, R. (2018). Application of adaptive Neuro-fuzzy inference system to estimate alongshore sediment transport rate (A real case study: southern shorelines of Caspian Sea). *Journal of Soft Computing in Civil Engineering*, 2(4), 72-85.
- Sagias, V. D., Zacharia, P., Tempeloudis, A., & Stergiou, C. (2024). Adaptive neuro-Fuzzy inference system-based predictive modeling of mechanical properties in additive manufacturing. *Machines*, 12(8), 523.
- Seber, G. A., & Wild, C. J. (2003). Nonlinear regression. hoboken. *New Jersey: John Wiley & Sons*, 62(63), 1238.
- Souza, P. V. d. C., Guimares, A. J., Rezende, T. S., Araujo, V. S., Araujo, V. J. S., & Batista, L. O. (2019). Bayesian fuzzy clustering neural network for regression problems. 2019 IEEE international conference on systems, man and cybernetics (SMC),
- Takagi, T., & Sugeno, M. (1985). Fuzzy identification of systems and its applications to modeling and control. *IEEE transactions on systems, man, and cybernetics*(1), 116-132.
- Thamaraimanalan, T., Gopal, D., Vignesh, S., & Kishore Kumar, K. (2025). Exploiting adaptive neuro-fuzzy inference systems for cognitive patterns in multimodal brain signal analysis. *Scientific reports*, 15(1), 9029.
- Wang, Y., Nonzee, N. J., Zhang, H., Ashing, K. T., Song, G., & Crespi, C. M. (2025). Interpretation of coefficients in segmented regression for interrupted time series analyses. *BMC Medical Research Methodology*, 25(1), 98.