

Multi-Attribute Medical Diagnosis Using Interval-Valued Spherical Fuzzy Prioritized Dombi–Hamy Mean Operators

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ABSTRACT

Decision-making (DM) information is a crucial and prevalent way to assess the majority of difficulties in real-life challenges, allowing one to determine the optimal choice from the collection of preferences. To prioritize aggregation operators (PAOs), the bundled data is first aggregated into a singleton set, which is calculated using algebraic laws. Hamy mean (HM) information is also employed. Proposing Dombi operational laws for Interval value spherical fuzzy (IVSF) information and attempting to build the theory of HM information based on Dombi t-norm (TN) and t-conorm (T-CN) while taking into account SF information are the main contributions of this theory. These operators are referred to as IVSF prioritized Dombi HM (IVSFPDHM), IVSF weighted prioritized Dombi HM (IVSFWPDHM), IVSF prioritized Dombi dual HM (IVSFPDDHM), and IVSF weighted prioritized Dombi dual HM (IVSFWPDDHM) operators. We also assessed a few important characteristics and findings for the data that were provided in the examined analysis. Additionally, this book uses MADM "multi-attribute decision-making" information based on pioneering operators and provides some examples to support the importance and dominance of the assessed information. In order to improve the quality of the generated operators, we lastly contrasted the assessed information with some other outdated or current information.

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1. Introduction

The main goal of MADM problems is to determine the most appropriate alternative from a limited set of alternatives by considering preference ratings given by experts or decision makers (DMs) with respect to multiple attributes. However, in real-world situations, DMs usually encounter complex and ambiguous environments (Sreedaran & Ponnuswamy, 2024), making it difficult to express their decisions in precise numerical form. Almost all phenomena in the real world involve some degree of uncertainty and inaccuracy, especially related to human opinion and subjective judgment. To cope with such vagueness, Zadeh (1975) introduced the concept of fuzzy sets (FSs), where each element is characterized by a membership degree (MD) representing its belongingness to a set. Later, Atanassov (1999) extended this idea by introducing a non-membership degree (NMD) and proposed intuitionistic fuzzy sets (IFSs). In an IFS, the sum of the MD and NMD must lie within the Zadeh (1975). Although this framework improves the representation of uncertainty, the imposed restriction limits the flexibility of DMs in assigning their preferred MD and NMD. Yager (2008) later introduced Pythagorean fuzzy sets (PEFS), where the constraint is relaxed by requiring that the sum of the squares of the MD and the NMD be within Zadeh (1975). This modification provides more flexibility than IFS. Later, the Q-Rung orthopair fuzzy set (Q-ROFS) was proposed as a further generalization, which allowed the sum q th power for MD and NMD to remain within Zadeh (1975), thus increasing the range of allowed information. In many real-world decision-making situations, especially those based on human judgment, the decision-maker may hesitate, refuse, or avoid giving a particular opinion, and ignoring this behavior may lead to a loss of information. To overcome this limitation, Cuong et al. (2016) introduced the Picture Fuzzy Set (PFS), which includes an additional avoidance degree with MD and NMD; the sum of these three values is bounded by Zadeh (1975). Later, this framework has been extended to spherical fuzzy sets (SFS), where the sum of

squares of MD, AD, and NMD is bounded

1.1 Literature Review

Selecting the best options has been a topic of great debate in MADM when it comes to information or data that is impacted by imprecision and uncertainty. Information gathering is the primary technique for MADM difficulties. Numerous TN and TCN serve as the foundation for different aggregation operators (AOs). The discourse surrounding arithmetic AOs is well-established, having been covered in many frameworks. These include IF arithmetic AOs, fuzzy number IF arithmetic AOs (Wang, 2008), and IF hybrid arithmetic AOs. Li and Xu (2019) gave the concept of prioritized AOs based on the priority degree in MADM.

Chen and Xu (2014) expand the prioritized AOs on the OWA operator and the prioritized measure. Perez-Arellano et al. (2020) gave the new concept of prioritized AOs based on Bonferroni. Akram et al. (2022) expand the concept of AOs under the complex SF prioritized weighted AOs. Akram et al. (2020) gave the concept of SF, prioritized weighted AOs based on the MADM. Ashraf et al. (2020) expand the theory of SF Dombi AOs based on the MADM. Riaz et al. (2020) gave the concept of q-ROFS prioritized AOs based on the MADM. Mahmood et al. (2021) gave the concept of Dombi, prioritized weighted AOs based on the MULTIMOORA method. Yan et al. (2011) expand the concept of prioritized weighted AOs on MADM. Wei and Wei (2018) prioritized weighted AOs on MADM. Sarfraz et al. (2022) gave the new concept of prioritized AOs based on the Aczel- Alsina (AA) TN and TM using the applications in group DM.

Among many other TN and TCN, Dombi TN and Dombi T-CN (Dombi, 1982) are also extensively explored, leading to the invention of Dombi AOs. Jana et al. (2019); Seikh and Mandal (2021) have introduced the concepts of DAOs in the PyFS environment and DAOs in the framework of IFSSs, respectively. Sarkar and Biswas (2021) have also examined issues impacted by QROFS DAOs. DAOs of PFSs have been introduced. He et al. (2019) gave the concept of DHM operators, their application on the basis of project assessment. Saad et al. (2024) expand the concept of DHM AOs on the basis of PF. Sinani et al. (2020) evaluated the rough DHM AOs. Mahmood and Ali (2023) expand the theory on complex IF based on the DHM operator. Akram et al. (2022) gave the concept of DHM AOs on the basis of the DM framework on the 2-tuple linguistic fuzzy. Deb et al. (2022) gave the concept of Dombi Bonferroni Mean operators and used the dual hesitant fuzzy operator on the basis of MADM. Huzaira Razzaque et al. (2023) expand the theory of Dombi, given the concept of Dombi AOs hepatitis diagnosis on the basis of SF AOs. Khan et al. (2021) gave the concept of SF Dombi AOs on the basis of MADM. Mandal and Seikh (2023) gave the concept of Dombi AOs on the basis of IVSF. Hussain and Pamucar (2022) gave the concept of prioritized AA AOs based on the complex IF information and using the application of Group DM.

Donyatalab et al. (2022) gave the concept of medical diagnosis based on SF distance and similarity measures. Riaz et al. (2021) gave the concept of SF based on the application of medical diagnosis. Rayappan and Krishnaswamy (2020) expand the concept of similarity measures of SF based on the application in medical diagnosis. Gong et al. (2016) introduce the application on a medical diagnosis base for IF analysis. Ye (2017) introduce the application of medical diagnosis using the m-polar SF. Ashraf et al. (2023) introduce the application for the diagnosis of COVID-19 based on the SF decision process. Hussain et al. (2023) gave the concept of medical diagnosis on the basis of MADM using the PyFS AOs.

1.2 Research Gap

All existing information works well under the restriction that the criteria and DM are at the same priority level. Because some researchers have derived several operators but cannot include the degree of priority, some individuals have derived many operators based on different criteria under the assessment of priority information, but cannot use the DHM criteria. Under the assumption of priority degree, achieving the theory of AO based on DHM TN and TCN is a very challenging task for the entire FS theory community.

Because in forming these ideas, we have needed to combine four different types of structures, which is a very complex and strange task. From these ideas, we can easily derive the works of Simple AO, PAO, and DHM AO, which were obtained in different manuscripts by four different researchers.

1.3 Contributions

The following is a list of this paper's contributions:

- We have proposed some new PAOs for SF, including IVSFPDHM, IVFPDHM, IVFWPDHM, and IVFWPDHM operators and discussed some related properties.
- We designed a new spherical fuzzy MADM method based on IVFWPDHM and IVFWPDDHM.
- By solving investment decision-related problems, we evaluated the applicability of our proposed aggregation function-based MADM method.
- Parameter analysis and comparison analysis are performed with the existing method to show the reliability and effectiveness of the proposed method.

1.4 Organization of the Study

Therefore, the structure of this paper is organized as follows. Part 2 presents a brief review of basic concepts related to SF and Dombi operations on SF numbers. In Section 3, several AOs, namely SFPDHM, SFPWDHM, SFPDDHM, and SFPWDDHM, are defined, and their essential properties are thoroughly investigated. Part 4 shows the practical application of the proposed IVSFN in skin cancer assessment. Section 5 provides a comparative analysis, highlighting the effectiveness of the proposed operators compared to existing methods. Finally, Section 6 concludes the paper by summarizing the main findings and emphasizing the study's importance.

2. Preliminaries

2.1. The fundamental ideas of IVIFS and IVSFS are presented in this section as an introduction to the suggested work. These ideas will aid in our comprehension of this content.

Definition 1(Khalil et al., 2019): Suppose X be a universe of discourse, an IVSF Q' over X is an object having the form as follows:

$$Q' = \{(x, \theta^2_{Q'}(x), \varpi^2_{Q'}(x), \cap^2_{Q'}(x)) | x \in X\}$$

Where $\theta^2_{Q'}(x) \subseteq [0, 1]$, $\varpi^2_{Q'}(x) \subseteq [0, 1]$ and $\cap^2_{Q'}(x)$ are interval numbers $0 \leq \sup(\theta^2_{Q'}(x)) + \sup(\varpi^2_{Q'}(x)) + \sup(\cap^2_{Q'}(x)) \leq 1, \forall x \in X$. For convenience, consider $\theta^2_{Q'}(x) = ([\theta^2, \Lambda^2], [\delta^2, \mu^2])$ and $\cap^2_{Q'}(x) = ([\cap^2, \varpi^2], [\theta^2, \Lambda^2], [\delta^2, \mu^2])$ is IVSFNs.

Definition 2 (Xu & Yager, 2008): Suppose $\cap' = ([\cap, \varpi], [\theta, \Lambda], [\delta, \mu])$ be an IVSPFN, a score capability S can be characterized as follows:

$$S(\cap) = \frac{1}{3}(2 + \cap^2 - \varpi^2 + \theta^2 - \Lambda^2 + \delta^2 - \mu^2), S(\cap') \in [-1, 1]$$

Definition 3 (Xu & Yager, 2008): Suppose $\cap' = ([\cap, \varpi], [\theta, \Lambda], [\delta, \mu])$ be an IVSPFN, a score capability H can be characterized as

$$H(\cap) = \frac{1}{3}(2 + \cap^2 + \varpi^2 + \theta^2 + \Lambda^2 + \delta^2 + \mu^2), H(\cap') \in [0, 1].$$

To assess the degree of accuracy of the IVSPFN $\cap' = ([\cap, \varpi], [\theta, \Lambda], [\delta, \mu])$

Definition 4 (Xu & Yager, 2008): $\cap_1 = ([\cap_1, \varpi_1], [\theta_1, \Lambda_1], [\delta_1, \mu_1])$ and $\cap_2 = ([\cap_2, \varpi_2], [\theta_2, \Lambda_2], [\delta_2, \mu_2])$ be three IVSFNs, $S(\cap_1) = \frac{1}{3}(2 + \cap_1^2 - \varpi_1^2 + \theta_1^2 - \Lambda_1^2 + \delta_1^2 - \mu_1^2)$ and $S(\cap_2) = \frac{1}{3}(2 + \cap_2^2 - \varpi_2^2 + \theta_2^2 - \Lambda_2^2 + \delta_2^2 - \mu_2^2)$ be the scores of $\cap'_1$ and $\cap'_2$ individually, and let $H(\cap_1) = \frac{1}{3}(2 + \cap_1^2 + \varpi_1^2 + \theta_1^2 + \Lambda_1^2 + \delta_1^2 + \mu_1^2)$ and $H(\cap_2) = \frac{1}{3}(2 + \cap_2^2 + \varpi_2^2 + \theta_2^2 + \Lambda_2^2 + \delta_2^2 + \mu_2^2)$ be the accuracy degrees of $\cap_1$ and $\cap_2$ respectively, then if $S(\cap_1) < S(\cap_2)$ then $(\cap_1) < (\cap'_2)$; if $S(\cap_1) < S(\cap_2)$ then (1) if $H(\cap_1) < H(\cap_2)$ then $(\cap'_1) < (\cap'_2)$; if $H(\cap'_1) < H(\cap'_2)$ then $(\cap'_1) < (\cap'_2)$.

Definition 5 (Xu & Yager, 2008): The two IVSFNs $\cap'_1 = ([\cap_1, \varpi_1], [\theta_1, \Lambda_1], [\delta_1, \mu_1])$ and $\cap'_2 = ([\cap_2, \varpi_2], [\theta_2, \Lambda_2], [\delta_2, \mu_2])$ the observing functional regulations are characterized as follows:

$$\begin{aligned} \cap'_1 \oplus \cap'_2 &= \left(\sqrt[2]{[\cap_1^2 + \cap_2^2 - \cap_1^2 \cap_2^2, \varpi_1^2 + \varpi_2^2 - \varpi_1^2 \varpi_2^2]}, \right. \\ &\quad \left. [\theta_1^2 \theta_2^2, \Lambda_1^2 \Lambda_2^2], [\delta_1^2 \delta_2^2, \mu_1^2 \mu_2^2] \right) \\ \cap'_1 \otimes \cap'_2 &= \left(\sqrt[2]{[\cap_1^2 \cap_2^2, \varpi_1^2 \varpi_2^2]}, \right. \\ &\quad \left. [\theta_1^2 + \theta_2^2 - \theta_1^2 \theta_2^2, \Lambda_1^2 + \Lambda_2^2 - \Lambda_1^2 \Lambda_2^2], \right. \\ &\quad \left. [\delta_1^2 + \delta_2^2 - \delta_1^2 \delta_2^2, \mu_1^2 + \mu_2^2 - \mu_1^2 \mu_2^2] \right) \\ \lambda \cap'_1 &= \left(\sqrt[2]{[1 - (1 - \cap_1^2)^\lambda, 1 - (1 - \varpi_1^2)^\lambda]}, [(\theta_1^2)^\lambda, (\Lambda_1^2)^\lambda], [(\delta_1^2)^\lambda, (\mu_1^2)^\lambda] \right), \lambda > 0 \\ \cap_1^{\lambda} &= \left(\sqrt[2]{[(\cap_1^2)^\lambda, (\varpi_1^2)^\lambda]}, [1 - (1 - \theta_1^2)^\lambda, 1 - (1 - \Lambda_1^2)^\lambda], \right. \\ &\quad \left. [1 - (1 - \delta_1^2)^\lambda, 1 - (1 - \mu_1^2)^\lambda] \right), \lambda > 0 \end{aligned}$$

2.1 HM Operator

The HM operator was suggested by Wu et al. (2018), In which using 2-tuple linguistic neutrosophic numbers (2TLNNs), we extend the HM, weighted Hamy mean (WHM), dual Hamy mean (DHM), and weighted dual Hamy mean (WDHM) operators in this study.

Definition 6 (Dombi, 1982): The HM operator is characterized as follows:

$$HM^{\Gamma} \cap'_1, \cap'_2, \dots, \cap'_v = \frac{\sum_{1 \leq i_1 < \dots < i_{\Gamma} \leq v} (\prod_{l=1}^{\Gamma} \cap'_{i_l})^{1/\Gamma}}{C_v^{\Gamma}} \quad (1)$$

Where Γ a parameter is $\Gamma = 1, 2, \dots, \mathfrak{L}$, $\mathfrak{o}_1, \mathfrak{o}_2, \dots, \mathfrak{o}_\Gamma$ are Γ whole number qualities taken from the set $\{1, 2, \dots, \mathfrak{L}\}$ of δ whole number qualities C_Ω^Γ is the binomial coefficient, $C_\Omega^\Gamma = \frac{\Omega!}{\Gamma!(\Omega-\Gamma)!}$.

2.1 Dombi Operations of IVSFNs

Definition 7(Dombi, 1982): Dombi proposed a generator to create Dombi T-N and T-CN, which are displayed as observes:

$$D(q, r) = \frac{1}{1 + \left(\left(\frac{1-q}{q} \right)^\lambda + \left(\frac{1-r}{r} \right)^\lambda \right)^{1/\lambda}}$$

$$D^c(q, r) = 1 - \frac{1}{1 + \left(\left(\frac{q}{1-q} \right)^\lambda + \left(\frac{r}{1-r} \right)^\lambda \right)^{1/\lambda}}$$

Where $\varpi > 0$, $(q, r) \in [0, 1]$. Based on the Dombi T-N and T-CN, we can give the functional rules of IVIFNs.

Definition 8: The two IVPFNs $\mathfrak{n}'_1 = ([\mathfrak{n}_1, \varpi_1], [\theta_1, \Lambda_1], [\delta_1, \mu_1])$ and $\mathfrak{n}'_2 = ([\mathfrak{n}_2, \varpi_2], [\theta_2, \Lambda_2], [\delta_2, \mu_2])$ $\varpi > 0$, the Dombi operational laws are characterized as follows:

$$\varphi'_1 \oplus \varphi'_2 = \left(\left[\begin{array}{l} 1 - \frac{1}{1 + \left(\left(\frac{\mathfrak{n}_1^2}{1-\mathfrak{n}_1^2} \right)^\lambda + \left(\frac{\mathfrak{n}_2^2}{1-\mathfrak{n}_2^2} \right)^\lambda \right)^{1/\lambda}}, 1 - \frac{1}{1 + \left(\left(\frac{\varpi_1^2}{1-\varpi_1^2} \right)^\lambda + \left(\frac{\varpi_2^2}{1-\varpi_2^2} \right)^\lambda \right)^{1/\lambda}} \end{array} \right], \left[\begin{array}{l} \frac{1}{1 + \left(\left(\frac{1-\theta_1^2}{\theta_1^2} \right)^\lambda + \left(\frac{1-\theta_2^2}{\theta_2^2} \right)^\lambda \right)^{1/\lambda}}, \frac{1}{1 + \left(\left(\frac{1-\Lambda_1^2}{\Lambda_1^2} \right)^\lambda + \left(\frac{1-\Lambda_2^2}{\Lambda_2^2} \right)^\lambda \right)^{1/\lambda}} \end{array} \right], \left[\begin{array}{l} \frac{1}{1 + \left(\left(\frac{1-\delta_1^2}{\delta_1^2} \right)^\lambda + \left(\frac{1-\delta_2^2}{\delta_2^2} \right)^\lambda \right)^{1/\lambda}}, \frac{1}{1 + \left(\left(\frac{1-\mu_1^2}{\mu_1^2} \right)^\lambda + \left(\frac{1-\mu_2^2}{\mu_2^2} \right)^\lambda \right)^{1/\lambda}} \end{array} \right] \right)$$

$$\mathfrak{n}'_1 \otimes \mathfrak{n}'_2 = \left(\left[\begin{array}{l} 1 - \frac{1}{1 + \left(\left(\frac{1-\mathfrak{n}_1^2}{\mathfrak{n}_1^2} \right)^\lambda + \left(\frac{1-\mathfrak{n}_2^2}{\mathfrak{n}_2^2} \right)^\lambda \right)^{1/\lambda}}, 1 - \frac{1}{1 + \left(\left(\frac{1-\varpi_1^2}{\varpi_1^2} \right)^\lambda + \left(\frac{1-\varpi_2^2}{\varpi_2^2} \right)^\lambda \right)^{1/\lambda}} \end{array} \right], \left[\begin{array}{l} \frac{1}{1 + \left(\left(\frac{\theta_1^2}{1-\theta_1^2} \right)^\lambda + \left(\frac{\theta_2^2}{1-\theta_2^2} \right)^\lambda \right)^{1/\lambda}}, \frac{1}{1 + \left(\left(\frac{\Lambda_1^2}{1-\Lambda_1^2} \right)^\lambda + \left(\frac{\Lambda_2^2}{1-\Lambda_2^2} \right)^\lambda \right)^{1/\lambda}} \end{array} \right], \left[\begin{array}{l} \frac{1}{1 + \left(\left(\frac{\delta_1^2}{1-\delta_1^2} \right)^\lambda + \left(\frac{\delta_2^2}{1-\delta_2^2} \right)^\lambda \right)^{1/\lambda}}, \frac{1}{1 + \left(\left(\frac{\mu_1^2}{1-\mu_1^2} \right)^\lambda + \left(\frac{\mu_2^2}{1-\mu_2^2} \right)^\lambda \right)^{1/\lambda}} \end{array} \right] \right)$$

$$n \cap'_1 = \left(\left[\begin{array}{c} 1 - \frac{1}{1 + \left(n \left(\frac{\cap_1^2}{1 - \cap_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(n \left(\frac{\varpi_1^2}{1 - \varpi_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ \frac{1}{1 + \left(n \left(\frac{1 - \theta_1^2}{\theta_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(n \left(\frac{1 - \lambda_1^2}{\lambda_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ \frac{1}{1 + \left(n \left(\frac{1 - \delta_1^2}{\delta_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(n \left(\frac{1 - \mu_1^2}{\mu_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right] \right)$$

$$(\cap'_1)^n = \left(\left[\begin{array}{c} \frac{1}{1 + \left(n \left(\frac{1 - \cap_1^2}{\cap_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(n \left(\frac{1 - \varpi_1^2}{\varpi_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ 1 - \frac{1}{1 + \left(n \left(\frac{\theta_1^2}{1 - \theta_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(n \left(\frac{\lambda_1^2}{1 - \lambda_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ 1 - \frac{1}{1 + \left(n \left(\frac{\delta_1^2}{1 - \delta_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(n \left(\frac{\mu_1^2}{1 - \mu_1^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{array} \right] \right)$$

3. IVSFNs are used by Some Prioritized Dombi Hamy Mean Operators

The IVSFPDHM Operator is defined as follows by the HM operator and Domi Operation rules:

Definition 9: Suppose $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \lambda_L], [\delta_L, \mu_L]) (\mathcal{L} = 1, 2, \dots, \mathfrak{L})$ be a set of IVSFNs. The IVSFPDHM operator is

$$IVSFPDHM^{(\Gamma)}(\varphi'_1, \varphi'_2, \dots, \varphi'_\mathfrak{L}) = \frac{\bigoplus_{1 \leq \varphi_1 < \dots < \varphi_\Gamma \leq \mathfrak{L}} \left(\bigotimes_{\mathcal{L}=1}^{\Gamma} \varphi'_{\varphi_\mathcal{L}} \right)^{\frac{1}{\Gamma}}}{c_\mathfrak{L}^\Gamma} \quad (2)$$

Theorem 1: Consider $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \lambda_L], [\delta_L, \mu_L]) (\mathcal{L} = 1, 2, \dots, \mathfrak{L})$ be a set of IVSFNs. The IVSFPDHM operator is also an IVSFN where

$$IVSFPDHM^{(\Gamma)}(\varphi'_1, \varphi'_2, \dots, \varphi'_\mathfrak{L}) = \frac{\bigoplus_{1 \leq \varphi_1 < \dots < \varphi_\Gamma \leq \mathfrak{L}} \left(\bigotimes_{\mathcal{L}=1}^{\Gamma} \varphi'_{\varphi_\mathcal{L}} \right)^{\frac{1}{\Gamma}}}{c_\mathfrak{L}^\Gamma}$$

$$= \left(\begin{array}{c} \left[\frac{1}{1 - \frac{1}{1 + \left[\frac{\frac{1}{c_{\mathcal{L}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_{\Gamma} \leq \mathcal{L}} \frac{1}{\sum_{L=1}^{\Gamma} \left(\frac{1 - \alpha_{o_L}^2}{\alpha_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}}, \frac{1}{1 - \frac{1}{1 + \left[\frac{\frac{1}{c_{\mathcal{L}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_{\Gamma} \leq \mathcal{L}} \frac{1}{\sum_{L=1}^{\Gamma} \left(\frac{1 - \omega_{o_L}^2}{\omega_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}}, \right. \\ \left[\frac{1}{1 + \left[\frac{\frac{1}{c_{\mathcal{L}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_{\Gamma} \leq \mathcal{L}} \frac{1}{\sum_{L=1}^{\Gamma} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left[\frac{\frac{1}{c_{\mathcal{L}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_{\Gamma} \leq \mathcal{L}} \frac{1}{\sum_{L=1}^{\Gamma} \left(\frac{\Lambda_{o_L}^2}{1 - \Lambda_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \right. \\ \left. \left[\frac{1}{1 + \left[\frac{\frac{1}{c_{\mathcal{L}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_{\Gamma} \leq \mathcal{L}} \frac{1}{\sum_{L=1}^{\Gamma} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left[\frac{\frac{1}{c_{\mathcal{L}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_{\Gamma} \leq \mathcal{L}} \frac{1}{\sum_{L=1}^{\Gamma} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}} \right] \end{array} \right)$$

Proof: Based on the DHM operation laws of IVSNs, there are:

$$\begin{aligned} \mathcal{L} = 1 \quad \bigotimes_{\mathcal{L} = 1}^{\Gamma} \quad \cap'_{o_{\mathcal{L}}} &= \left(\begin{array}{c} \left[\frac{1}{1 + \left(\sum_{L=1}^{\Gamma} \left(\frac{1 - \alpha_{o_L}^2}{\alpha_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} \left(\frac{1 - \omega_{o_L}^2}{\omega_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \right. \\ \left[\frac{1}{1 + \left(\sum_{L=1}^{\Gamma} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} \left(\frac{\Lambda_{o_L}^2}{1 - \Lambda_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \right. \\ \left. \left[\frac{1}{1 + \left(\sum_{L=1}^{\Gamma} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \right] \end{array} \right) \\ \left(\bigotimes_{\mathcal{L} = 1}^{\Gamma} \quad \cap'_{o_{\mathcal{L}}} \right)^{\frac{1}{\Gamma}} &= \left(\begin{array}{c} \left[\frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} \left(\frac{1 - \alpha_{o_L}^2}{\alpha_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} \left(\frac{1 - \omega_{o_L}^2}{\omega_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \right. \\ \left[\frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} \left(\frac{\Lambda_{o_L}^2}{1 - \Lambda_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \right. \\ \left. \left[\frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \right] \end{array} \right) \end{aligned}$$

$$\oplus_{1 \leq o_1 < \dots < o_L \leq v} \left(\bigotimes_{L=1}^{\Gamma} \cap'_{o_L} \right)^{\frac{1}{\Gamma}} = \left(\begin{array}{c} \left[1 - \frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{1 - \alpha_{o_L}^2}{\alpha_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{1 - \omega_{o_L}^2}{\omega_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{\lambda_{o_L}^2}{1 - \lambda_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \end{array} \right)^{\frac{1}{\Gamma}}$$

$$IVSFPDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_v) = \frac{\oplus_{1 \leq o_1 < \dots < o_L \leq v} \left(\bigotimes_{L=1}^{\Gamma} \cap'_{o_L} \right)^{\frac{1}{\Gamma}}}{c_v^{\Gamma}}$$

$$= \left(\begin{array}{c} \left[1 - \frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{1 - \alpha_{o_L}^2}{\alpha_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{1 - \omega_{o_L}^2}{\omega_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{\lambda_{o_L}^2}{1 - \lambda_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\Gamma}{\sum_{L=1}^{\Gamma} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \end{array} \right)$$

Example1: Suppose $\cap'_1 = ([0.4, 0.6], [0.3, 0.5], [0.1, 0.4])$, $\cap'_2 = ([0.3, 0.5], [0.5, 0.7], [0.2, 0.4])$, $\cap'_3 = ([0.2, 0.4], [0.4, 0.5], [0.4, 0.6])$, and $\cap'_4 = ([0.4, 0.5], [0.3, 0.6], [0.5, 0.6])$ be four IVSFNs and $\Gamma = 2, \lambda = 3$,

$$\frac{1-\alpha_{\vartheta_L}^2}{\alpha_{\vartheta_L}^2} = (5.2500, 10.111, 24.00, 5.250)$$

$$\frac{1-\theta_{\vartheta_L}^2}{\theta_{\vartheta_L}^2} = (10.111, 3.0000, 5.2500, 10.111)$$

$$\frac{1-\delta_{\vartheta_L}^2}{\delta_{\vartheta_L}^2} = (99.000, 24.000, 3.0000, 3.0000)$$

$$\frac{\varpi_{\vartheta_L}^2}{1-\varpi_{\vartheta_L}^2} = (0.5625, 0.3333, 0.1905, 0.5625)$$

$$\frac{\Lambda_{\vartheta_L}^2}{1-\Lambda_{\vartheta_L}^2} = (0.3333, 0.9608, 0.3333, 0.19047)$$

$$\frac{\mu_{\vartheta_L}^2}{1-\mu_{\vartheta_L}^2} = (0.1905, 0.1905, 0.5625, 0.5625)$$

$$IVSFPDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) = \frac{\bigoplus_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \left(\bigotimes_{\ell=1}^{\Gamma} \cap'_{\vartheta_\ell} \right)^{\frac{1}{\Gamma}}}{c_\vartheta^\Gamma}$$

$$= \left[\begin{array}{c} 1 - \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(5.2500)^3 + (10.111)^3} + \frac{1}{(5.2500)^3 + (24.00)^3} + \frac{1}{(5.2500)^3 + (5.250)^3} \right) + \frac{1}{(10.111)^3 + (24.00)^3} + \frac{1}{(10.111)^3 + (5.250)^3} + \frac{1}{(24.00)^3 + (5.250)^3} \right) \right)^{\frac{1}{3}}}, \\ 1 / \left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(10.111)^3 + (3.000)^3} + \frac{1}{(10.111)^3 + (5.2500)^3} + \frac{1}{(10.111)^3 + (10.111)^3} \right) + \frac{1}{(3.0000)^3 + (5.2500)^3} + \frac{1}{(3.0000)^3 + (10.111)^3} + \frac{1}{(5.2500)^3 + (10.111)^3} \right) \right)^{\frac{1}{3}} \right) \\ \left[\begin{array}{c} 1 - \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(99.000)^3 + (24.000)^3} + \frac{1}{(99.000)^3 + (3.0000)^3} + \frac{1}{(99.000)^3 + (3.0000)^3} \right) + \frac{1}{(24.000)^3 + (3.0000)^3} + \frac{1}{(24.000)^3 + (3.0000)^3} + \frac{1}{(3.0000)^3 + (3.0000)^3} \right) \right)^{\frac{1}{3}}}, \\ \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(0.5625)^3 + (0.3333)^3} + \frac{1}{(0.5625)^3 + (0.1905)^3} + \frac{1}{(0.5625)^3 + (0.5625)^3} \right) + \frac{1}{(0.3333)^3 + (0.1905)^3} + \frac{1}{(0.3333)^3 + (0.5625)^3} + \frac{1}{(0.1905)^3 + (0.5625)^3} \right) \right)^{\frac{1}{3}} \right)} \end{array} \right] \\ \left[\begin{array}{c} 1 - \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(0.3333)^3 + (0.9608)^3} + \frac{1}{(0.3333)^3 + (0.3333)^3} + \frac{1}{(0.3333)^3 + (0.1904)^3} \right) + \frac{1}{(0.9608)^3 + (0.3333)^3} + \frac{1}{(0.9608)^3 + (0.1904)^3} + \frac{1}{(0.3333)^3 + (0.1904)^3} \right) \right)^{\frac{1}{3}}}, \\ 1 / \left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(0.1905)^3 + (0.1905)^3} + \frac{1}{(0.1905)^3 + (0.5625)^3} + \frac{1}{(0.1905)^3 + (0.5625)^3} \right) + \frac{1}{(0.1905)^3 + (0.5625)^3} + \frac{1}{(0.1905)^3 + (0.5625)^3} + \frac{1}{(0.5625)^3 + (0.5625)^3} \right) \right)^{\frac{1}{3}} \right) \end{array} \right] \\ = [0.0201, 0.8676][0.9750, 0.1064][0.9424, 0.0616] \end{array}$$

Property 1: (Idempotency) If $\cap_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])(\mathcal{L} = 1, 2, \dots, \vartheta) = \cap$ are equal, then

$$IVSFPDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) = \cap$$

Property 2: (Monotonicity) If $\cap_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])(\mathcal{L} = 1, 2, \dots, \vartheta)$ and $\theta'_L = ([r_L^2, p_L^2], [m_L^2, l_L^2], [s_L^2, d_L^2])(\mathcal{L} = 1, 2, \dots, \vartheta)$ be two sets of IVSFNs. If $\cap_L^2 \leq r_L^2$, $\theta_L^2 \leq m_L^2$, $\delta_L^2 \leq s_L^2$ and $\varpi_L^2 \geq$

$p_{\mathcal{L}}^2, \Lambda_{\mathcal{L}}^2 \geq l_{\mathcal{L}}^2, \mu_{\mathcal{L}}^2 \geq d_{\mathcal{L}}^2$ hold for all $\mathcal{L}$, then

$$IVSFPDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_Q) \leq IVSFPDHM^{(\Gamma)}(\theta'_1, \theta'_2, \dots, \theta'_Q)$$

Property 3: (Boundedness) Suppose $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L]) (\mathcal{L} = 1, 2, \dots, Q)$ be a set of IVSFNs. If $\cap'^+_{\circ} = \left(\begin{bmatrix} \max_{\circ}(\cap_L^2), \max_{\circ}(\theta_L^2), \max_{\circ}(\delta_L^2) \end{bmatrix}, \begin{bmatrix} \min_{\circ}(\varpi_L^2), \min_{\circ}(\Lambda_L^2), \min_{\circ}(\mu_L^2) \end{bmatrix} \right)$ and If $\cap'^-_{\circ} = \left(\begin{bmatrix} \min_{\circ}(\cap_L^2), \min_{\circ}(\theta_L^2), \min_{\circ}(\delta_L^2) \end{bmatrix}, \begin{bmatrix} \max_{\circ}(\varpi_L^2), \max_{\circ}(\Lambda_L^2), \max_{\circ}(\mu_L^2) \end{bmatrix} \right)$ then

$$\cap'^- \leq IVSFPDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_Q) \leq \cap'^+$$

3.1 The IVFWPDHM Operator

It is crucial to pay attention to attribute weights in a real MADM. The interval valued spherical fuzzy weighted prioritized Dombi Hamy mean (IVSFWPDHM) operator is what we suggest as a result.

Definition 10: Suppose $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L]) (\mathcal{L} = 1, 2, \dots, Q)$ be a set of IVSFNs with their weight vector $\mathcal{W}_{\circ} = (\mathcal{W}_1, \mathcal{W}_2, \dots, \mathcal{W}_Q)^T$, thereby satisfying $\mathcal{W}_{\circ} \in [0, 1]$ and $\sum_{\circ=1}^Q \mathcal{W}_{\circ} = 1$. Then the IVSFWPDHM operator is as follows:

$$IVSFWPDHM_{\mathcal{W}}^{\Gamma}(\cap'_1, \cap'_2, \dots, \cap'_Q) = \frac{\oplus_{1 \leq \circ_1 < \dots < \circ_{\Gamma} \leq Q} \left(\bigotimes_{\mathcal{L}=1}^{\Gamma} ((\cap'_{\circ_{\mathcal{L}}})^{\mathcal{W}_{\circ_{\mathcal{L}}}}) \right)^{\frac{1}{\Gamma}}}{c_{\mathcal{Q}}^{\Gamma}} \quad (3)$$

Theorem 2: Suppose $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L]) (\mathcal{L} = 1, 2, \dots, Q) (\mathcal{L} = 1, 2, \dots, Q)$ be a set of IVSFNs. The IVSFWPDHM operator is also an IVSFN where

$$IVSFWPDHM_{\mathcal{W}}^{\Gamma}(\cap'_1, \cap'_2, \dots, \cap'_Q) = \frac{\oplus_{1 \leq \circ_1 < \dots < \circ_{\Gamma} \leq Q} \left(\bigotimes_{\mathcal{L}=1}^{\Gamma} ((\cap'_{\circ_{\mathcal{L}}})^{\mathcal{W}_{\circ_{\mathcal{L}}}}) \right)^{\frac{1}{\Gamma}}}{c_{\mathcal{Q}}^{\Gamma}} = \left(\begin{bmatrix} 1 - \frac{1}{1 + \left[\frac{1}{c_{\mathcal{Q}}^{\Gamma}} \sum_{1 \leq \circ_1 < \dots < \circ_{\Gamma} \leq Q} \frac{\Gamma}{\sum_{\mathcal{L}=1}^{\Gamma} \mathcal{W}_{\circ_{\mathcal{L}}} \left(\frac{1 - \Delta_{\circ_{\mathcal{L}}}^2}{\cap_{\circ_{\mathcal{L}}}^2} \right)^{\frac{1}{\Lambda}} \right]} \right)^{\frac{1}{\Lambda}}, 1 - \frac{1}{1 + \left[\frac{1}{c_{\mathcal{Q}}^{\Gamma}} \sum_{1 \leq \circ_1 < \dots < \circ_{\Gamma} \leq Q} \frac{\Gamma}{\sum_{\mathcal{L}=1}^{\Gamma} \mathcal{W}_{\circ_{\mathcal{L}}} \left(\frac{1 - \varpi_{\circ_{\mathcal{L}}}^2}{\varpi_{\circ_{\mathcal{L}}}^2} \right)^{\frac{1}{\Lambda}} \right]} \right)^{\frac{1}{\Lambda}}, \end{bmatrix} \right. \\ \left. \begin{bmatrix} \frac{1}{1 + \left[\frac{1}{c_{\mathcal{Q}}^{\Gamma}} \sum_{1 \leq \circ_1 < \dots < \circ_{\Gamma} \leq Q} \frac{\Gamma}{\sum_{\mathcal{L}=1}^{\Gamma} \mathcal{W}_{\circ_{\mathcal{L}}} \left(\frac{\theta_{\circ_{\mathcal{L}}}^2}{1 - \theta_{\circ_{\mathcal{L}}}^2} \right)^{\frac{1}{\Lambda}} \right]} \right)^{\frac{1}{\Lambda}}, \frac{1}{1 + \left[\frac{1}{c_{\mathcal{Q}}^{\Gamma}} \sum_{1 \leq \circ_1 < \dots < \circ_{\Gamma} \leq Q} \frac{\Gamma}{\sum_{\mathcal{L}=1}^{\Gamma} \mathcal{W}_{\circ_{\mathcal{L}}} \left(\frac{\Lambda_{\circ_{\mathcal{L}}}^2}{1 - \Lambda_{\circ_{\mathcal{L}}}^2} \right)^{\frac{1}{\Lambda}} \right]} \right)^{\frac{1}{\Lambda}}, \end{bmatrix} \right. \\ \left. \begin{bmatrix} \frac{1}{1 + \left[\frac{1}{c_{\mathcal{Q}}^{\Gamma}} \sum_{1 \leq \circ_1 < \dots < \circ_{\Gamma} \leq Q} \frac{\Gamma}{\sum_{\mathcal{L}=1}^{\Gamma} \mathcal{W}_{\circ_{\mathcal{L}}} \left(\frac{\delta_{\circ_{\mathcal{L}}}^2}{1 - \delta_{\circ_{\mathcal{L}}}^2} \right)^{\frac{1}{\Lambda}} \right]} \right)^{\frac{1}{\Lambda}}, \frac{1}{1 + \left[\frac{1}{c_{\mathcal{Q}}^{\Gamma}} \sum_{1 \leq \circ_1 < \dots < \circ_{\Gamma} \leq Q} \frac{\Gamma}{\sum_{\mathcal{L}=1}^{\Gamma} \mathcal{W}_{\circ_{\mathcal{L}}} \left(\frac{\mu_{\circ_{\mathcal{L}}}^2}{1 - \mu_{\circ_{\mathcal{L}}}^2} \right)^{\frac{1}{\Lambda}} \right]} \right)^{\frac{1}{\Lambda}}, \end{bmatrix} \right)$$

Proof: Based on the DHM operation laws of IVSNs, there are:

$$\begin{aligned}
 (\cap'_{o_L})^{w_{o_L}} &= \left(\begin{bmatrix} \frac{1}{1 + \left(w_{o_L} \left(\frac{1 - \cap_{o_L}^2}{\cap_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(w_{o_L} \left(\frac{1 - \varpi_{o_L}^2}{\varpi_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ \frac{1 - \frac{1}{1 + \left(w_{o_L} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(w_{o_L} \left(\frac{\Lambda_{o_L}^2}{1 - \Lambda_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ \frac{1 - \frac{1}{1 + \left(w_{o_L} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(w_{o_L} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{bmatrix} \right) \\
 \bigotimes_{L=1}^{\Gamma} (\cap'_{o_L})^{w_{o_L}} &= \left(\begin{bmatrix} \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{1 - \cap_{o_L}^2}{\cap_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{1 - \varpi_{o_L}^2}{\varpi_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ \frac{1 - \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\Lambda_{o_L}^2}{1 - \Lambda_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ \frac{1 - \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{bmatrix} \right) \\
 \left(\bigotimes_{L=1}^{\Gamma} (\cap'_{o_L})^{w_{o_L}} \right)^{\frac{1}{\Gamma}} &= \left(\begin{bmatrix} \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{1 - \cap_{o_L}^2}{\cap_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{1 - \varpi_{o_L}^2}{\varpi_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ \frac{1 - \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\Lambda_{o_L}^2}{1 - \Lambda_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \\ \frac{1 - \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \end{bmatrix} \right)
 \end{aligned}$$

$$\oplus_{1 \leq o_1 < \dots < o_L \leq \vartheta} \left(\bigotimes_{L=1}^{\Gamma} (\cap'_{o_L})^{w_{o_L}} \right)^{\frac{1}{\Gamma}} = \left(\begin{array}{c} \left[\frac{1 - \frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{1 - \cap_{o_L}^2}{\cap_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}{1}, \frac{1 - \frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{1 - \varpi_{o_L}^2}{\varpi_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}{1} \right] \\ \left[\frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\lambda_{o_L}^2}{1 - \lambda_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \end{array} \right)^{\frac{1}{\Gamma}}$$

$$IVSFWDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) = \frac{\oplus_{1 \leq o_1 < \dots < o_L \leq \vartheta} \left(\bigotimes_{L=1}^{\Gamma} (\cap'_{o_L})^{w_{o_L}} \right)^{\frac{1}{\Gamma}}}{c_{\vartheta}^{\Gamma}}$$

$$= \left(\begin{array}{c} \left[\frac{1 - \frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{1 - \cap_{o_L}^2}{\cap_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}{1}, \frac{1 - \frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{1 - \varpi_{o_L}^2}{\varpi_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}{1} \right] \\ \left[\frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\theta_{o_L}^2}{1 - \theta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\lambda_{o_L}^2}{1 - \lambda_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\delta_{o_L}^2}{1 - \delta_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{\left[1 + \frac{1}{\sum_{L=1}^{\Gamma} w_{o_L} \left(\frac{\mu_{o_L}^2}{1 - \mu_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \end{array} \right)^{\frac{1}{\Gamma}}$$

Example2: Suppose $\cap'_1 = ([0.4, 0.6], [0.3, 0.5], [0.1, 0.4])$ $\cap'_2 = ([0.3, 0.5], [0.5, 0.7], [0.2, 0.4])$, $\cap'_3 = ([0.2, 0.4], [0.4, 0.5], [0.4, 0.6])$, and $\cap'_4 = ([0.4, 0.5], [0.3, 0.6], [0.5, 0.6])$ be four IVSFNs and $\Gamma = 2, \lambda = 3$,

$$\frac{1 - \cap_{\circ_L}^2}{\cap_{\circ_L}^2} = (5.2500, 10.111, 24.00, 5.250)$$

$$\frac{1 - \theta_{\circ_L}^2}{\theta_{\circ_L}^2} = (10.111, 3.0000, 5.2500, 10.111)$$

$$\frac{1 - \delta_{\circ_L}^2}{\delta_{\circ_L}^2} = (99.000, 24.000, 3.0000, 3.0000)$$

$$\frac{\varpi_{\circ_L}^2}{1 - \varpi_{\circ_L}^2} = (0.5625, 0.3333, 0.1905, 0.5625)$$

$$\frac{\Lambda_{\circ_L}^2}{1 - \Lambda_{\circ_L}^2} = (0.3333, 0.9608, 0.3333, 0.19047)$$

$$\frac{\mu_{\circ_L}^2}{1 - \mu_{\circ_L}^2} = (0.1905, 0.1905, 0.5625, 0.5625)$$

$$IVSFWPDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) = \frac{\bigoplus_{1 \leq \circ_1 < \dots < \circ_\Gamma \leq \vartheta} \left(\begin{matrix} \Gamma \\ \mathcal{L} = 1 \end{matrix} \otimes \cap'_{\circ_L} \right)^{\frac{1}{\Gamma}}}{\mathcal{C}_\vartheta^\Gamma}$$

$$\left[\begin{array}{c} 1 - \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{\frac{1}{0.4450 \times (5.2500)^3 + 0.3060 \times (10.111)^3 + 0.4450 \times (5.2500)^3 + 0.1644 \times (24.00)^3 + 0.4450 \times (5.2500)^3 + 0.0845 \times (5.250)^3}}{\frac{1}{0.3060 \times (10.111)^3 + 0.1644 \times (24.00)^3 + 0.3060 \times (10.111)^3 + 0.0845 \times (5.250)^3 + 0.1644 \times (24.00)^3 + 0.0845 \times (5.250)^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}}, \\ 1 / \left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{\frac{1}{0.4450 \times (10.111)^3 + 0.3060 \times (3.000)^3} + \frac{1}{0.4450 \times (10.111)^3 + 0.1644 \times (5.2500)^3} + \frac{1}{0.4450 \times (10.111)^3 + 0.0845 \times (10.111)^3}}{\frac{1}{0.3060 \times (3.0000)^3 + 0.1644 \times (5.2500)^3} + \frac{1}{0.3060 \times (3.0000)^3 + 0.0845 \times (10.111)^3} + \frac{1}{0.1644 \times (5.2500)^3 + 0.0845 \times (10.111)^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}} \\ \left[\begin{array}{c} 1 - \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{\frac{1}{0.4450 \times (99.000)^3 + 0.3060 \times (24.000)^3 + 0.4450 \times (99.000)^3 + 0.1644 \times (3.0000)^3 + 0.4450 \times (99.000)^3 + 0.0845 \times (3.0000)^3}}{\frac{1}{0.3060 \times (24.000)^3 + 0.1644 \times (3.0000)^3 + 0.3060 \times (24.000)^3 + 0.0845 \times (3.0000)^3 + 0.1644 \times (3.0000)^3 + 0.0845 \times (3.0000)^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}}, \\ \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{\frac{1}{0.4450 \times (0.5625)^3 + 0.3060 \times (0.3333)^3 + 0.4450 \times (0.5625)^3 + 0.1644 \times (0.1905)^3 + 0.4450 \times (0.5625)^3 + 0.0845 \times (0.5625)^3}}{\frac{1}{0.3060 \times (0.3333)^3 + 0.1644 \times (0.1905)^3 + 0.3060 \times (0.3333)^3 + 0.0845 \times (0.5625)^3 + 0.1644 \times (0.1905)^3 + 0.0845 \times (0.5625)^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}} \\ 1 - \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{\frac{1}{0.4450 \times (0.3333)^3 + 0.3060 \times (0.9608)^3 + 0.4450 \times (0.3333)^3 + 0.1644 \times (0.3333)^3 + 0.4450 \times (0.3333)^3 + 0.0845 \times (0.1904)^3}}{\frac{1}{0.3060 \times (0.9608)^3 + 0.1644 \times (0.3333)^3 + 0.3060 \times (0.9608)^3 + 0.0845 \times (0.1904)^3 + 0.1644 \times (0.3333)^3 + 0.0845 \times (0.1904)^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}}, \\ 1 / \left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{\frac{1}{0.4450 \times (0.1905)^3 + 0.3060 \times (0.1905)^3} + \frac{1}{0.4450 \times (0.1905)^3 + 0.1644 \times (0.5625)^3} + \frac{1}{0.4450 \times (0.1905)^3 + 0.0845 \times (0.5625)^3}}{\frac{1}{0.3060 \times (0.1905)^3 + 0.1644 \times (0.5625)^3} + \frac{1}{0.3060 \times (0.1905)^3 + 0.0845 \times (0.5625)^3} + \frac{1}{0.1644 \times (0.5625)^3 + 0.0845 \times (0.5625)^3}} \right)^{\frac{1}{3}} \right) \right)^{\frac{1}{3}} \end{array} \right] \\ = [0.0003, 0.9972][0.9996, 0.0020][0.9968, 0.0004]$$

Property 4: (Monotonicity) If $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])$ ($L = 1, 2, \dots, \vartheta$) and $\theta'_L = ([r_L^2, p_L^2], [m_L^2, l_L^2], [s_L^2, d_L^2])$ ($L = 1, 2, \dots, \vartheta$) be two sets of IVSFNs. If $\cap_L^2 \leq r_L^2$, $\theta_L^2 \leq m_L^2$, $\delta_L^2 \leq s_L^2$ and $\varpi_L^2 \geq p_L^2$, $\Lambda_L^2 \geq l_L^2$, $\mu_L^2 \geq d_L^2$ hold for all L , then

$$IVSFWPDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) \leq IVSFWPDHM^{(\Gamma)}(\theta'_1, \theta'_2, \dots, \theta'_\vartheta)$$

Property 5: (Boundedness) Suppose $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])$ ($L = 1, 2, \dots, \vartheta$) be a set of IVSFNs. If $\cap'_L{}^+ = ([ma\Gamma_{\circ}(\cap_L^2), ma\Gamma_{\circ}(\theta_L^2), ma\Gamma_{\circ}(\delta_L^2)], [min_{\circ}(\cap_L^2), min_{\circ}(\theta_L^2), min_{\circ}(\delta_L^2)])$, and If $\cap'_L{}^- = ([ma\Gamma_{\circ}(\varpi_L^2), ma\Gamma_{\circ}(\Lambda_L^2), ma\Gamma_{\circ}(\mu_L^2)], [min_{\circ}(\varpi_L^2), min_{\circ}(\Lambda_L^2), min_{\circ}(\mu_L^2)])$

then

$$\cap'^- \leq IVSFWPDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) \leq \cap'^+$$

3.2 The IVSFPDDHM Operator

The IVSFWPDHM Operator Wu et al. (2018) proposed the dual HM (DHM) operator.

Definition 11(Wu et al., 2018): The DHM operator is as follows:

$$DHM^{(\Gamma)}(\varphi_1, \varphi_2, \dots, \varphi_\vartheta) = \left(\prod_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \left(\frac{\sum_{\ell=1}^{\Gamma} \varphi_{\vartheta_\ell}}{\Gamma} \right) \right)^{\frac{1}{C_\vartheta^\Gamma}} \quad (4)$$

Where Γ is a parameter and $\Gamma = 1, 2, \dots, \vartheta, \vartheta_1, \vartheta_2, \dots, \vartheta_\Gamma$ are Γ integer values taken from the set $\{1, 2, \dots, \vartheta\}$ of k integer values, C_ϑ^Γ denotes the binomial coefficient and $C_\vartheta^\Gamma = \frac{\vartheta!}{\Gamma!(\vartheta-\Gamma)!}$.

In this section, we will propose the interval values spherical fuzzy prioritized dual Dombi DHM (IVSFPDDHM) operator.

Definition 12: Suppose $\cap'_\mathcal{L} = ([\cap_\mathcal{L}, \varpi_\mathcal{L}], [\theta_\mathcal{L}, \lambda_\mathcal{L}], [\delta_\mathcal{L}, \mu_\mathcal{L}]) (\mathcal{L} = 1, 2, \dots, \vartheta)$ be a set of IVSFNs. The IVSFPDDHM operator is

$$IVSFPDDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) = \left(\bigotimes_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \left(\frac{\bigoplus_{\ell=1}^{\Gamma} \cap'_{\vartheta_\ell}}{\Gamma} \right) \right)^{\frac{1}{C_\vartheta^\Gamma}} \quad (5)$$

Theorem 3: Suppose $\cap'_\mathcal{L} = ([\cap_\mathcal{L}, \varpi_\mathcal{L}], [\theta_\mathcal{L}, \lambda_\mathcal{L}], [\delta_\mathcal{L}, \mu_\mathcal{L}]) (\mathcal{L} = 1, 2, \dots, \vartheta)$ be a set of IVSFNs. The IVSFPDDHM operator is also an IVSFN where

$$IVSFPDDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) = \left(\bigotimes_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \left(\frac{\bigoplus_{\ell=1}^{\Gamma} \cap'_{\vartheta_\ell}}{\Gamma} \right) \right)^{\frac{1}{C_\vartheta^\Gamma}} \\ = \left(\begin{array}{c} \left[\frac{1}{1 + \left[\frac{\frac{1}{C_\vartheta^\Gamma} \sum_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} \left(\frac{\cap_{\vartheta_\ell}^2}{1 - \cap_{\vartheta_\ell}^2} \right)^\lambda} \right]^\lambda}, \frac{1}{1 + \left[\frac{\frac{1}{C_\vartheta^\Gamma} \sum_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} \left(\frac{\varpi_{\vartheta_\ell}^2}{1 - \varpi_{\vartheta_\ell}^2} \right)^\lambda} \right]^\lambda} \right]^\lambda, \\ \left[1 - \frac{1}{1 + \left[\frac{\frac{1}{C_\vartheta^\Gamma} \sum_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} \left(\frac{1 - \theta_{\vartheta_\ell}^2}{\theta_{\vartheta_\ell}^2} \right)^\lambda} \right]^\lambda}, 1 - \frac{1}{1 + \left[\frac{\frac{1}{C_\vartheta^\Gamma} \sum_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} \left(\frac{1 - \lambda_{\vartheta_\ell}^2}{\lambda_{\vartheta_\ell}^2} \right)^\lambda} \right]^\lambda} \right]^\lambda, \\ \left[1 - \frac{1}{1 + \left[\frac{\frac{1}{C_\vartheta^\Gamma} \sum_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} \left(\frac{1 - \delta_{\vartheta_\ell}^2}{\delta_{\vartheta_\ell}^2} \right)^\lambda} \right]^\lambda}, 1 - \frac{1}{1 + \left[\frac{\frac{1}{C_\vartheta^\Gamma} \sum_{1 \leq \vartheta_1 < \dots < \vartheta_\Gamma \leq \vartheta} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} \left(\frac{1 - \mu_{\vartheta_\ell}^2}{\mu_{\vartheta_\ell}^2} \right)^\lambda} \right]^\lambda} \right]^\lambda \end{array} \right)$$

Proof: Based on the DHM operation laws of IVSNs, there are:

$$\begin{aligned}
 & \bigoplus_{\mathcal{L}=\mathbb{1}}^{\Gamma} \cap'_{\mathfrak{o}_{\mathcal{L}}} = \left(\begin{array}{c} \left[\frac{1}{1 + \left(\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{\mathfrak{o}_{\mathcal{L}}^2}{1 - \mathfrak{o}_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{\varpi_{\mathcal{L}}^2}{1 - \varpi_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left(\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \theta_{\mathcal{L}}^2}{\theta_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \Lambda_{\mathcal{L}}^2}{\Lambda_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left(\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \delta_{\mathcal{L}}^2}{\delta_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \mu_{\mathcal{L}}^2}{\mu_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \right] \end{array} \right) \\
 & \frac{\bigoplus_{\mathcal{L}=\mathbb{1}}^{\Gamma} \cap'_{\mathfrak{o}_{\mathcal{L}}}}{\Gamma} = \left(\begin{array}{c} \left[\frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{\mathfrak{o}_{\mathcal{L}}^2}{1 - \mathfrak{o}_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{\varpi_{\mathcal{L}}^2}{1 - \varpi_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \theta_{\mathcal{L}}^2}{\theta_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \Lambda_{\mathcal{L}}^2}{\Lambda_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \delta_{\mathcal{L}}^2}{\delta_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{\Gamma} \sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \mu_{\mathcal{L}}^2}{\mu_{\mathcal{L}}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}} \right] \end{array} \right) \\
 & \bigotimes_{\mathbb{1} \leq \mathfrak{o}_{\mathbb{1}} < \dots < \mathfrak{o}_{\Gamma} \leq \mathfrak{O}} \left(\frac{\bigoplus_{\mathcal{L}=\mathbb{1}}^{\Gamma} \cap'_{\mathfrak{o}_{\mathcal{L}}}}{\Gamma} \right) = \left(\begin{array}{c} \left[\frac{1}{1 + \left[\frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{\mathfrak{o}_{\mathcal{L}}^2}{1 - \mathfrak{o}_{\mathcal{L}}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{\varpi_{\mathcal{L}}^2}{1 - \varpi_{\mathcal{L}}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left[\frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \theta_{\mathcal{L}}^2}{\theta_{\mathcal{L}}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \Lambda_{\mathcal{L}}^2}{\Lambda_{\mathcal{L}}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left[\frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \delta_{\mathcal{L}}^2}{\delta_{\mathcal{L}}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \mu_{\mathcal{L}}^2}{\mu_{\mathcal{L}}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}} \right] \end{array} \right)
 \end{aligned}$$

$$IVSFPDDHM^{(\Gamma)}(\varphi'_1, \varphi'_2, \dots, \varphi'_\Omega) = \left(\bigotimes_{1 \leq \nu_1 < \dots < \nu_\Gamma \leq \Omega} \left(\frac{\frac{\Gamma}{\mathcal{L}=\mathbb{1}} \oplus \varphi'_{\nu_\Gamma}}{\Gamma} \right) \right)^{\frac{1}{\mathcal{C}_\Omega^\Gamma}}$$

$$= \left(\begin{array}{c} \left[\frac{1}{1 + \left[\frac{1}{\mathcal{C}_\Omega^\Gamma} \sum_{1 \leq \nu_1 < \dots < \nu_\Gamma \leq \Omega} \frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{\mathcal{C}_{\nu_\Gamma}^2}{1 - \mathcal{C}_{\nu_\Gamma}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{1}{\mathcal{C}_\Omega^\Gamma} \sum_{1 \leq \nu_1 < \dots < \nu_\Gamma \leq \Omega} \frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{\mathcal{C}_{\nu_\Gamma}^2}{1 - \mathcal{C}_{\nu_\Gamma}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \\ \left[1 - \frac{1}{1 + \left[\frac{1}{\mathcal{C}_\Omega^\Gamma} \sum_{1 \leq \nu_1 < \dots < \nu_\Gamma \leq \Omega} \frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \theta_{\nu_\Gamma}^2}{\theta_{\nu_\Gamma}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left[\frac{1}{\mathcal{C}_\Omega^\Gamma} \sum_{1 \leq \nu_1 < \dots < \nu_\Gamma \leq \Omega} \frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \Lambda_{\nu_\Gamma}^2}{\Lambda_{\nu_\Gamma}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \\ \left[1 - \frac{1}{1 + \left[\frac{1}{\mathcal{C}_\Omega^\Gamma} \sum_{1 \leq \nu_1 < \dots < \nu_\Gamma \leq \Omega} \frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \delta_{\nu_\Gamma}^2}{\delta_{\nu_\Gamma}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left[\frac{1}{\mathcal{C}_\Omega^\Gamma} \sum_{1 \leq \nu_1 < \dots < \nu_\Gamma \leq \Omega} \frac{\Gamma}{\sum_{\mathcal{L}=\mathbb{1}}^{\Gamma} \left(\frac{1 - \mu_{\nu_\Gamma}^2}{\mu_{\nu_\Gamma}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \end{array} \right)^{\frac{1}{\mathcal{C}_\Omega^\Gamma}}$$

Example3: Suppose $\mathcal{C}'_1 = ([0.4, 0.6], [0.3, 0.5], [0.1, 0.4])$, $\mathcal{C}'_2 = ([0.3, 0.5], [0.5, 0.7], [0.2, 0.4])$, $\mathcal{C}'_3 = ([0.2, 0.4], [0.4, 0.5], [0.4, 0.6])$, and $\mathcal{C}'_4 = ([0.4, 0.5], [0.3, 0.6], [0.5, 0.6])$ be four IVSFNs and $\Gamma = 2, \lambda = 3$,

$$\frac{\mathcal{C}_{\nu_\Gamma}^2}{1 - \mathcal{C}_{\nu_\Gamma}^2} = (1.7778, 3.0000, 5.2500, 1.7778)$$

$$\frac{\theta_{\nu_\Gamma}^2}{1 - \theta_{\nu_\Gamma}^2} = (3.0000, 1.0408, 3.0000, 5.2500)$$

$$\frac{\delta_{\nu_\Gamma}^2}{1 - \delta_{\nu_\Gamma}^2} = (5.2500, 5.2500, 1.7778, 1.7778)$$

$$\frac{1 - \mathcal{C}_{\nu_\Gamma}^2}{\mathcal{C}_{\nu_\Gamma}^2} = (0.1905, 0.0989, 0.0417, 0.1905)$$

$$\frac{1 - \Lambda_{\nu_\Gamma}^2}{\Lambda_{\nu_\Gamma}^2} = (0.0989, 0.3333, 0.19047, 0.0989)$$

$$\frac{1 - \mu_{\nu_\Gamma}^2}{\mu_{\nu_\Gamma}^2} = (0.0101, 0.0417, 0.3333, 0.3333)$$

$$IVSFPDDHM^{(\Gamma)}(\mathcal{C}'_1, \mathcal{C}'_2, \dots, \mathcal{C}'_\Omega) = \frac{\bigoplus_{1 \leq \nu_1 < \dots < \nu_\Gamma \leq \Omega} \left(\frac{\Gamma}{\mathcal{L}=\mathbb{1}} \bigotimes \mathcal{C}'_{\nu_\Gamma} \right)}{\mathcal{C}_\Omega^\Gamma}$$

$$= \left[\begin{array}{c} \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(1.7778)^3 + (3.0000)^3} + \frac{1}{(1.7778)^3 + (5.2500)^3} + \frac{1}{(1.7778)^3 + (1.7778)^3} \right) \right)^{\frac{1}{3}} \right)}, \\ 1 - 1 / \left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(3.0000)^3 + (1.0408)^3} + \frac{1}{(3.0000)^3 + (3.0000)^3} + \frac{1}{(3.0000)^3 + (5.2500)^3} \right) \right)^{\frac{1}{3}} \right)}, \\ \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(5.2500)^3 + (5.2500)^3} + \frac{1}{(5.2500)^3 + (1.7778)^3} + \frac{1}{(5.2500)^3 + (1.7778)^3} \right) \right)^{\frac{1}{3}} \right)}, \\ 1 - \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(0.1905)^3 + (0.0989)^3} + \frac{1}{(0.1905)^3 + (0.0417)^3} + \frac{1}{(0.1905)^3 + (0.1905)^3} \right) \right)^{\frac{1}{3}} \right)}, \\ \frac{1}{\left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(0.0989)^3 + (0.3333)^3} + \frac{1}{(0.0989)^3 + (0.19047)^3} + \frac{1}{(0.0989)^3 + (0.0989)^3} \right) \right)^{\frac{1}{3}} \right)}, \\ 1 - 1 / \left(1 + \left(\frac{2}{c_4^2} \times \left(\frac{1}{(0.0101)^3 + (0.0417)^3} + \frac{1}{(0.0101)^3 + (0.3333)^3} + \frac{1}{(0.0101)^3 + (0.3333)^3} \right) \right)^{\frac{1}{3}} \right)} \end{array} \right] \\ = [0.1552, 0.9042][0.8511, 0.9864][0.0170, 0.0025]$$

Property 6: (*Idempotency*) If $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])(L = 1, 2, \dots, \mathfrak{L}) = \cap$ are equal, then $IVSFPDDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\mathfrak{L}) = \cap$

Property 7: (*Monotonicity*) If $\cap'_L = ([\cap_L^2, \varpi_L^2], [\theta_L^2, \Lambda_L^2], [\delta_L^2, \mu_L^2])(L = 1, 2, \dots, \mathfrak{L})$ and $\theta'_L = ([r_L^2, p_L^2], [m_L^2, l_L^2], [s_L^2, d_L^2])(L = 1, 2, \dots, \mathfrak{L})$ be two sets of IVSFNs. If $\cap_L^2 \leq r_L^2, \theta_L^2 \leq m_L^2, \delta_L^2 \leq s_L^2$ and $\varpi_L^2 \geq p_L^2, \Lambda_L^2 \geq l_L^2, \mu_L^2 \geq d_L^2$ hold for all L , then

$$IVSFPDDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\mathfrak{L}) \leq IVSFPDDHM^{(\Gamma)}(\theta'_1, \theta'_2, \dots, \theta'_\mathfrak{L})$$

Property 8: (*Boundedness*) Suppose $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])(L = 1, 2, \dots, \mathfrak{L})$ be a set of IVSFNs. If $\cap'^+_{\mathfrak{o}} = ([ma\Gamma_{\mathfrak{o}}(\cap_L^2), ma\Gamma_{\mathfrak{o}}(\theta_L^2), ma\Gamma_{\mathfrak{o}}(\delta_L^2)], [mi2_{\mathfrak{o}}(\cap_L^2), mi2_{\mathfrak{o}}(\theta_L^2), mi2_{\mathfrak{o}}(\delta_L^2)])$ and If $\cap'^-_{\mathfrak{o}} = ([ma\Gamma_{\mathfrak{o}}(\varpi_L^2), ma\Gamma_{\mathfrak{o}}(\Lambda_L^2), ma\Gamma_{\mathfrak{o}}(\mu_L^2)], [mi2_{\mathfrak{o}}(\varpi_L^2), mi2_{\mathfrak{o}}(\Lambda_L^2), mi2_{\mathfrak{o}}(\mu_L^2)])$ then

$$\cap'^- \leq IVSFPDDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\mathfrak{L}) \leq \cap'^+$$

3.3 The IVSFWPDDHM Operator

In practical MADM, it is important to pay attention to assigning weights. We propose interval valued spherical fuzzy weighted priority duality Dombi WPDDHM (IVFWPDDHM) operator.

Definition 13: Suppose $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])(L = 1, 2, \dots, \mathfrak{L})$ be a set of IVSFNs with their weight vector $\mathcal{W}_{\mathfrak{o}} = (\mathcal{W}_1, \mathcal{W}_2, \dots, \mathcal{W}_{\mathfrak{L}})^T$, thereby satisfying $\mathcal{W}_{\mathfrak{o}} \in [0, 1]$ and $\sum_{\mathfrak{o}=1}^{\mathfrak{L}} \mathcal{W}_{\mathfrak{o}} = 1$. Then the IVSFWPDDHM operator is as follows:

$$IVSFPDDHM_{\mathcal{W}}^{\Gamma}(\phi'_1, \phi'_2, \dots, \phi'_{\mathfrak{L}}) = \left(\bigotimes_{1 \leq \mathfrak{o}_1 < \dots < \mathfrak{o}_{\Gamma} \leq \mathfrak{L}} \left(\frac{\bigoplus_{L=1}^{\Gamma} \mathcal{W}_{\mathfrak{o}_L} \phi'_{\mathfrak{o}_L}}{\Gamma} \right)^{\frac{1}{c_{\mathfrak{L}}^{\Gamma}}} \right) \quad (6)$$

Theorem 4: Suppose $\cap'_L = ([\cap_L, \varpi_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])(L = 1, 2, \dots, \mathfrak{L})$ be a set of IVSFNs. The IVSFWPDDHM operator is also an IVSFN where

$$IVSFPDDHM_{\mathcal{W}}^{\Gamma}(\varphi'_1, \varphi'_2, \dots, \varphi'_L) = \left(\bigotimes_{1 \leq o_1 < \dots < o_L \leq L} \left(\frac{\bigoplus_{\ell=1}^{\Gamma} w_{o_L} \varphi'_{o_L}}{\Gamma} \right)^{\frac{1}{c_{\mathcal{W}}^{\Gamma}}}} \right)$$

$$= \left(\left[\begin{array}{cc} \frac{1}{1 + \left[\frac{\frac{1}{c_{\mathcal{W}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_L \leq L} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{\rho_{o_L}^2}{1 - \rho_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left[\frac{\frac{1}{c_{\mathcal{W}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_L \leq L} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{\varpi_{o_L}^2}{1 - \varpi_{o_L}^2} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}} \right] \right. \\ \left. \frac{1}{1 - \left[\frac{\frac{1}{c_{\mathcal{W}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_L \leq L} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{1 - \theta_{o_L}}{\theta_{o_L}} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{1 - \left[\frac{\frac{1}{c_{\mathcal{W}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_L \leq L} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{1 - \Lambda_{o_L}}{\Lambda_{o_L}} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}} \right] \right. \\ \left. \frac{1}{1 - \left[\frac{\frac{1}{c_{\mathcal{W}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_L \leq L} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{1 - \delta_{o_L}}{\delta_{o_L}} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}}, \frac{1}{1 - \left[\frac{\frac{1}{c_{\mathcal{W}}^{\Gamma}} \sum_{1 \leq o_1 < \dots < o_L \leq L} \frac{\Gamma}{\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{1 - \mu_{o_L}}{\mu_{o_L}} \right)^{\lambda}} \right]^{\frac{1}{\lambda}}}} \right] \right)$$

Proof: Based on the DHM operation laws of IVSNs, there are:

$$w_{o_L} \cap'_{o_L} = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(w_{o_L} \left(\frac{\rho_{o_L}^2}{1 - \rho_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(w_{o_L} \left(\frac{\varpi_{o_L}^2}{1 - \varpi_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}} \right] \right. \\ \left[\begin{array}{cc} \frac{1}{1 + \left(w_{o_L} \left(\frac{1 - \theta_{o_L}}{\theta_{o_L}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(w_{o_L} \left(\frac{1 - \Lambda_{o_L}}{\Lambda_{o_L}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}} \right] \\ \left[\begin{array}{cc} \frac{1}{1 + \left(w_{o_L} \left(\frac{1 - \delta_{o_L}}{\delta_{o_L}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(w_{o_L} \left(\frac{1 - \mu_{o_L}}{\mu_{o_L}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}} \right] \end{array} \right)$$

$$\bigoplus_{\ell=1}^{\Gamma} w_{o_L} \cap'_{o_L} = \left(\left[\begin{array}{cc} \frac{1}{1 + \left(\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{\rho_{o_L}^2}{1 - \rho_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{\varpi_{o_L}^2}{1 - \varpi_{o_L}^2} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}} \right] \right. \\ \left[\begin{array}{cc} \frac{1}{1 + \left(\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{1 - \theta_{o_L}}{\theta_{o_L}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{1 - \Lambda_{o_L}}{\Lambda_{o_L}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}} \right] \\ \left[\begin{array}{cc} \frac{1}{1 + \left(\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{1 - \delta_{o_L}}{\delta_{o_L}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}}, \frac{1}{1 + \left(\sum_{\ell=1}^{\Gamma} w_{o_L} \left(\frac{1 - \mu_{o_L}}{\mu_{o_L}} \right)^{\lambda} \right)^{\frac{1}{\lambda}}}} \right] \end{array} \right)$$

$$\begin{aligned}
 & \frac{\frac{r}{\ell=1} \oplus w_{o_L} \cap'_{o_L}}{r} = \left(\begin{array}{c} \left[1 - \frac{1}{1 + \left(\frac{1}{r} \sum_{L=1}^r w_{o_L} \left(\frac{\alpha_{o_L}^2}{1 - \alpha_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left(\frac{1}{r} \sum_{L=1}^r w_{o_L} \left(\frac{\varpi_{o_L}^2}{1 - \varpi_{o_L}^2} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left(\frac{1}{r} \sum_{L=1}^r w_{o_L} \left(\frac{1 - \theta_{o_L}}{\theta_{o_L}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{r} \sum_{L=1}^r w_{o_L} \left(\frac{1 - \lambda_{o_L}}{\lambda_{o_L}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \\ \left[\frac{1}{1 + \left(\frac{1}{r} \sum_{L=1}^r w_{o_L} \left(\frac{1 - \delta_{o_L}}{\delta_{o_L}} \right)^\lambda \right)^{\frac{1}{\lambda}}}, \frac{1}{1 + \left(\frac{1}{r} \sum_{L=1}^r w_{o_L} \left(\frac{1 - \mu_{o_L}}{\mu_{o_L}} \right)^\lambda \right)^{\frac{1}{\lambda}}} \right] \end{array} \right) \\
 & \otimes_{1 \leq o_1 < \dots < o_r \leq \varrho} \left(\frac{\frac{r}{\ell=1} \oplus w_{o_L} \cap'_{o_L}}{r} \right) = \left(\begin{array}{c} \left[\frac{1}{1 + \left[\frac{\frac{r}{\ell=1} \oplus w_{o_L} \left(\frac{\alpha_{o_L}^2}{1 - \alpha_{o_L}^2} \right)^\lambda}{\sum_{L=1}^r w_{o_L} \left(\frac{\alpha_{o_L}^2}{1 - \alpha_{o_L}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\frac{r}{\ell=1} \oplus w_{o_L} \left(\frac{\varpi_{o_L}^2}{1 - \varpi_{o_L}^2} \right)^\lambda}{\sum_{L=1}^r w_{o_L} \left(\frac{\varpi_{o_L}^2}{1 - \varpi_{o_L}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \\ \left[1 - \frac{1}{1 + \left[\frac{\frac{r}{\ell=1} \oplus w_{o_L} \left(\frac{1 - \theta_{o_L}}{\theta_{o_L}} \right)^\lambda}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \theta_{o_L}}{\theta_{o_L}} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left[\frac{\frac{r}{\ell=1} \oplus w_{o_L} \left(\frac{1 - \lambda_{o_L}}{\lambda_{o_L}} \right)^\lambda}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \lambda_{o_L}}{\lambda_{o_L}} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \\ \left[1 - \frac{1}{1 + \left[\frac{\frac{r}{\ell=1} \oplus w_{o_L} \left(\frac{1 - \delta_{o_L}}{\delta_{o_L}} \right)^\lambda}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \delta_{o_L}}{\delta_{o_L}} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left[\frac{\frac{r}{\ell=1} \oplus w_{o_L} \left(\frac{1 - \mu_{o_L}}{\mu_{o_L}} \right)^\lambda}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \mu_{o_L}}{\mu_{o_L}} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \end{array} \right) \\
 & IVSPDDHM^{(r)}(\varphi'_1, \varphi'_2, \dots, \varphi'_\varrho) = \left(\otimes_{1 \leq o_1 < \dots < o_r \leq \varrho} \left(\frac{\frac{r}{\ell=1} \oplus w_{o_L} \varphi'_{o_L}}{r} \right) \right)^{\frac{1}{c_\varrho^r}} \\
 & = \left(\begin{array}{c} \left[\frac{1}{1 + \left[\frac{\frac{1}{c_\varrho^r} \sum_{1 \leq o_1 < \dots < o_r \leq \varrho} \frac{r}{\sum_{L=1}^r w_{o_L} \left(\frac{\alpha_{o_L}^2}{1 - \alpha_{o_L}^2} \right)^\lambda}}{\sum_{L=1}^r w_{o_L} \left(\frac{\alpha_{o_L}^2}{1 - \alpha_{o_L}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, \frac{1}{1 + \left[\frac{\frac{1}{c_\varrho^r} \sum_{1 \leq o_1 < \dots < o_r \leq \varrho} \frac{r}{\sum_{L=1}^r w_{o_L} \left(\frac{\varpi_{o_L}^2}{1 - \varpi_{o_L}^2} \right)^\lambda}}{\sum_{L=1}^r w_{o_L} \left(\frac{\varpi_{o_L}^2}{1 - \varpi_{o_L}^2} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \\ \left[1 - \frac{1}{1 + \left[\frac{\frac{1}{c_\varrho^r} \sum_{1 \leq o_1 < \dots < o_r \leq \varrho} \frac{r}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \theta_{o_L}}{\theta_{o_L}} \right)^\lambda}}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \theta_{o_L}}{\theta_{o_L}} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left[\frac{\frac{1}{c_\varrho^r} \sum_{1 \leq o_1 < \dots < o_r \leq \varrho} \frac{r}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \lambda_{o_L}}{\lambda_{o_L}} \right)^\lambda}}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \lambda_{o_L}}{\lambda_{o_L}} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \\ \left[1 - \frac{1}{1 + \left[\frac{\frac{1}{c_\varrho^r} \sum_{1 \leq o_1 < \dots < o_r \leq \varrho} \frac{r}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \delta_{o_L}}{\delta_{o_L}} \right)^\lambda}}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \delta_{o_L}}{\delta_{o_L}} \right)^\lambda} \right]^{\frac{1}{\lambda}}}, 1 - \frac{1}{1 + \left[\frac{\frac{1}{c_\varrho^r} \sum_{1 \leq o_1 < \dots < o_r \leq \varrho} \frac{r}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \mu_{o_L}}{\mu_{o_L}} \right)^\lambda}}{\sum_{L=1}^r w_{o_L} \left(\frac{1 - \mu_{o_L}}{\mu_{o_L}} \right)^\lambda} \right]^{\frac{1}{\lambda}}} \right] \end{array} \right)
 \end{aligned}$$

Example 4: Suppose $\cap'_1 = ([0.4, 0.6], [0.3, 0.5], [0.1, 0.4])$ $\cap'_2 = ([0.3, 0.5], [0.5, 0.7], [0.2, 0.4])$, $\cap'_3 = ([0.2, 0.4], [0.4, 0.5], [0.4, 0.6])$, and $\cap'_4 = ([0.4, 0.5], [0.3, 0.6], [0.5, 0.6])$ be four IVSFNs and $\Gamma = 2, \lambda = 3$,

$$\frac{\cap_{\theta_L}^2}{1 - \cap_{\theta_L}^2} = (1.7778, 3.0000, 5.2500, 1.7778)$$

$$\frac{\theta_{\theta_L}^2}{1 - \theta_{\theta_L}^2} = (3.0000, 1.0408, 3.0000, 5.2500)$$

$$\frac{\delta_{\theta_L}^2}{1 - \delta_{\theta_L}^2} = (5.2500, 5.2500, 1.7778, 1.7778)$$

$$\frac{1 - \omega_{\theta_L}^2}{\omega_{\theta_L}^2} = (0.1905, 0.0989, 0.0417, 0.1905)$$

$$\frac{1 - \Lambda_{\theta_L}^2}{\Lambda_{\theta_L}^2} = (0.0989, 0.3333, 0.19047, 0.0989)$$

$$\frac{1 - \mu_{\theta_L}^2}{\mu_{\theta_L}^2} = (0.0101, 0.0417, 0.3333, 0.3333)$$

$$IVSFPDDWHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) = \frac{\bigoplus_{1 \leq \theta_1 < \dots < \theta_\Gamma \leq \vartheta} \left(\cap'_\theta \otimes \cap'_{\theta_L} \right)^{\frac{1}{\Gamma}}}{C_\vartheta^\Gamma}$$

$$\left[\begin{aligned} & \frac{1}{\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4749 \times (1.7778)^3 + 0.3182 \times (3.0000)^3 + 0.4749 \times (1.7778)^3 + 0.1485 \times (5.2500)^3 + 0.4749 \times (1.7778)^3 + 0.0584 \times (1.7778)^3}}{\frac{1}{0.3182 \times (3.0000)^3 + 0.1485 \times (5.2500)^3 + 0.3182 \times (3.0000)^3 + 0.0584 \times (1.7778)^3 + 0.1485 \times (5.2500)^3 + 0.0584 \times (1.7778)^3}} \right)^{\frac{1}{3}} \right)}^{\frac{1}{3}}, \\ & 1 - \frac{1}{\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4749 \times (3.0000)^3 + 0.3182 \times (1.0408)^3} + \frac{1}{0.4749 \times (3.0000)^3 + 0.1485 \times (3.0000)^3} + \frac{1}{0.4749 \times (3.0000)^3 + 0.0584 \times (5.2500)^3}}{\frac{1}{0.3182 \times (1.0408)^3 + 0.1485 \times (3.0000)^3} + \frac{1}{0.3182 \times (1.0408)^3 + 0.0584 \times (5.2500)^3} + \frac{1}{0.1485 \times (3.0000)^3 + 0.0584 \times (5.2500)^3}} \right)^{\frac{1}{3}} \right)}^{\frac{1}{3}}, \\ & \frac{1}{\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4749 \times (5.2500)^3 + 0.3182 \times (5.2500)^3} + \frac{1}{0.4749 \times (5.2500)^3 + 0.1485 \times (1.7778)^3} + \frac{1}{0.4749 \times (5.2500)^3 + 0.0584 \times (1.7778)^3}}{\frac{1}{0.3182 \times (5.2500)^3 + 0.1485 \times (1.7778)^3} + \frac{1}{0.3182 \times (5.2500)^3 + 0.0584 \times (1.7778)^3} + \frac{1}{0.1485 \times (1.7778)^3 + 0.0584 \times (1.7778)^3}} \right)^{\frac{1}{3}} \right)}^{\frac{1}{3}}, \\ & 1 - \frac{1}{\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4749 \times (0.1905)^3 + 0.3182 \times (0.0989)^3} + \frac{1}{0.4749 \times (0.1905)^3 + 0.1485 \times (0.0417)^3} + \frac{1}{0.4749 \times (0.1905)^3 + 0.0584 \times (0.1905)^3}}{\frac{1}{0.3182 \times (0.0989)^3 + 0.1485 \times (0.0417)^3} + \frac{1}{0.3182 \times (0.0989)^3 + 0.0584 \times (0.1905)^3} + \frac{1}{0.1485 \times (0.0417)^3 + 0.0584 \times (0.1905)^3}} \right)^{\frac{1}{3}} \right)}^{\frac{1}{3}}, \\ & \frac{1}{\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4749 \times (0.0989)^3 + 0.3182 \times (0.3333)^3} + \frac{1}{0.4749 \times (0.0989)^3 + 0.1485 \times (0.19047)^3} + \frac{1}{0.4749 \times (0.0989)^3 + 0.0584 \times (0.0989)^3}}{\frac{1}{0.3182 \times (0.3333)^3 + 0.1485 \times (0.19047)^3} + \frac{1}{0.3182 \times (0.3333)^3 + 0.0584 \times (0.0989)^3} + \frac{1}{0.1485 \times (0.19047)^3 + 0.0584 \times (0.0989)^3}} \right)^{\frac{1}{3}} \right)}^{\frac{1}{3}}, \\ & 1 - \frac{1}{\left(1 + \left(\frac{2}{C_4^2} \times \left(\frac{\frac{1}{0.4749 \times (0.0101)^3 + 0.3182 \times (0.0417)^3} + \frac{1}{0.4749 \times (0.0101)^3 + 0.1485 \times (0.3333)^3} + \frac{1}{0.4749 \times (0.0101)^3 + 0.0584 \times (0.3333)^3}}{\frac{1}{0.3182 \times (0.0417)^3 + 0.1485 \times (0.3333)^3} + \frac{1}{0.3182 \times (0.0417)^3 + 0.0584 \times (0.3333)^3} + \frac{1}{0.1485 \times (0.3333)^3 + 0.0584 \times (0.3333)^3}} \right)^{\frac{1}{3}} \right)}^{\frac{1}{3}} \end{aligned} \right]$$

$$= [0.2211, 0.8500][0.7292, 0.9911][0.0108, 0.0017]$$

Property 9: (*Monotonicity*) If $\cap'_L = ([\cap_L, \omega_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])(\mathcal{L} = 1, 2, \dots, \vartheta)$ and $\theta'_L = ([r_L^2, p_L^2], [m_L^2, l_L^2], [s_L^2, d_L^2])(\mathcal{L} = 1, 2, \dots, \vartheta)$ be two sets of IVSFNs. If $\cap_L^2 \leq r_L^2, \theta_L^2 \leq m_L^2, \delta_L^2 \leq s_L^2$ and $\omega_L^2 \geq p_L^2, \Lambda_L^2 \geq l_L^2, \mu_L^2 \geq d_L^2$ hold for all $\mathcal{L}$, then

$$IVSFWPDDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) \leq IVSFWPDDHM^{(\Gamma)}(\theta'_1, \theta'_2, \dots, \theta'_\vartheta)$$

Property 10: (*Boundedness*) Suppose $\cap'_L = ([\cap_L, \omega_L], [\theta_L, \Lambda_L], [\delta_L, \mu_L])(\mathcal{L} = 1, 2, \dots, \vartheta)$ be a set of IVSFNs. If $\cap'_o^+ = ([ma\Gamma_o(\cap_L^2), ma\Gamma_o(\theta_L^2), ma\Gamma_o(\delta_L^2)], [mi2_o(\cap_L^2), mi2_o(\theta_L^2), mi2_o(\delta_L^2)])$ and $\cap'_o^- = ([ma\Gamma_o(\omega_L^2), ma\Gamma_o(\Lambda_L^2), ma\Gamma_o(\mu_L^2)], [mi2_o(\omega_L^2), mi2_o(\Lambda_L^2), mi2_o(\mu_L^2)])$ then

$$\cap'^- \leq IVSFWPDDHM^{(\Gamma)}(\cap'_1, \cap'_2, \dots, \cap'_\vartheta) \leq \cap'^+$$

4. MADM Based on the SFDHM and SFDDHM Aggregation Operators

Consider a discrete set of alternatives $\mathcal{Q} = \{E_1, E_2, \dots, E_m\}$ being evaluated through a finite collection of attributes denoted as $\mathcal{E} = \{\bar{E}_1, \bar{E}_2, \dots, \bar{E}_n\}$ that fulfils the condition of prioritization. Let us presume that $\mathcal{W} = (\mathcal{W}_1, \mathcal{W}_2, \dots, \mathcal{W}_n)^T$ is the related weight vector of attributes subjected to the constraint $\mathcal{W}_j \in [0, 1]$ with $\sum_{j=1}^n \mathcal{W}_j = 1$. Each alternative $\mathcal{Q}_i (i = 1, 2, \dots, m)$ is assessed regarding the attributes $\bar{\mathcal{E}}_j (j = 1, 2, \dots, n)$ by an expert. The assessment values are provided in the form of SFVs, characterized as $\bar{\mathcal{L}}_{ij} = ([\cap_{\mathcal{L}}, \varpi_{\mathcal{L}}], [\theta_{\mathcal{L}}, \Lambda_{\mathcal{L}}], [\delta_{\mathcal{L}}, \mu_{\mathcal{L}}])$. These values measure how the alternative $\mathcal{Q}_i$ satisfies the attribute $\bar{\mathcal{E}}_j$ fulfilling the constraints $0 \leq (\cap_{\mathcal{L}})^2 + (\bar{\theta}_{\mathcal{L}})^2 + (\bar{\delta}_{\mathcal{L}})^2 \leq 1$ and $0 \leq (\varpi_{\mathcal{L}})^2 + (\bar{\Lambda}_{\mathcal{L}})^2 + (\bar{\mu}_{\mathcal{L}})^2 \leq 1$.

Taking these considerations into account, the detailed step-by-step computational procedure of the proposed MADM framework is outlined as follows:

4.1. Decision Algorithm

Step 1: The expert assessments for each alternative, related to its specific attributes, are organized in the SF decision matrix shown below.

$$M = [\bar{\mathcal{L}}_{ij}]_{m \times n} = ([\cap_{\mathcal{L}}, \varpi_{\mathcal{L}}], [\theta_{\mathcal{L}}, \Lambda_{\mathcal{L}}], [\delta_{\mathcal{L}}, \mu_{\mathcal{L}}])_{m \times n}. \quad (4.1)$$

Step 2: Within the context of MADM paradigms, attributes are characteristically classified into two types: benefit-type and cost-type. To ensure comparability and eliminate the effect of differing attribute natures, it is crucial to normalize all attributes by converting them into a consistent form. The normalized decision matrix, symbolized as $N = [\bar{\bar{\mathcal{L}}}_{ij}]_{m \times n}$, is attained using the subsequent formula:

$$\bar{\bar{\mathcal{L}}}_{ij} = \begin{cases} \bar{\mathcal{L}}_{ij} = ([\cap_{\mathcal{L}}, \varpi_{\mathcal{L}}], [\theta_{\mathcal{L}}, \Lambda_{\mathcal{L}}], [\delta_{\mathcal{L}}, \mu_{\mathcal{L}}]); & \text{for benefit attribute} \\ \bar{\mathcal{L}}_{ij}^c = ([\delta_{\mathcal{L}}, \mu_{\mathcal{L}}], [\theta_{\mathcal{L}}, \Lambda_{\mathcal{L}}], [\cap_{\mathcal{L}}, \varpi_{\mathcal{L}}]); & \text{for cost attribute.} \end{cases} \quad (4.2)$$

Step 3: Compile the entire set of data and calculate the aggregated SFVs of $\bar{\bar{\mathcal{L}}}_{ij}$ related to $\mathcal{Q}_i (i = 1, 2, \dots, m)$ concerning attributes $\bar{\mathcal{E}}_j (j = 1, 2, \dots, n)$ using the IVSFPDHM, IVSFWPDHM, IVSFPDDHM, and IVSFWPDDHM operators.

Step 4: Use definition 2. to calculate the score values of the aggregated SFVs attained in Step 4.

Step 5: Rank the alternatives $\mathcal{Q}_i (i = 1, 2, \dots, m)$ in descending order based on their calculated score values, and classify the most preferred alternative as the optimal candidate.

The procedural flow of the proposed MADM scheme is portrayed in Figure 1.

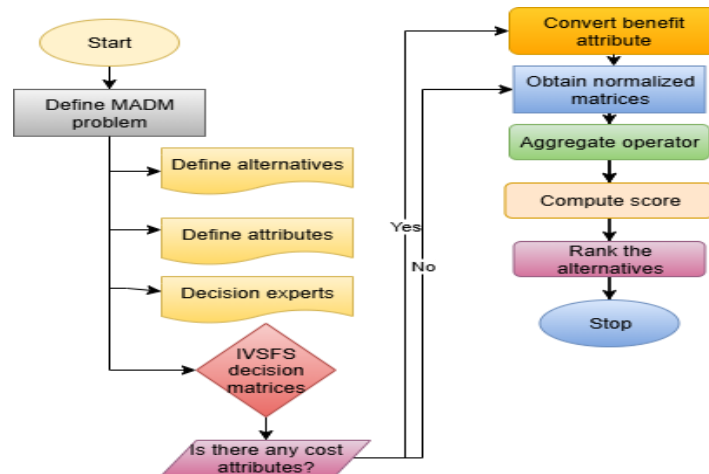


Figure 1: Flowchart of the proposed MADM method

4.2 Example and Comparison

In this section, we present a numerical example, compare the result with different parameters, and compare the result in precise aggregation operators.

4.3 Numerical Example

In this section, we present numerical examples based on coverage of skin cancer, and which we think could be used

when we enter in medical diagnosis.

4.4 Skin Cancer

Skin cancer affects both adults and children frequently. The main reason for skin cancers is exposure to ultraviolet (UV) radiation. UV radiation is a form of energy that comes from the sun and artificial sources like tanning beds. When the skin is exposed to UV radiation, it can cause damage to the DNA in skin cells, leading to the development of skin cancer. There are three main types of skin cancer, each associated with different types of skin cells and varying degrees of severity. It's important to take preventive measures to reduce exposure to UV radiation, such as using sunscreen, wearing protective clothing, and avoiding tanning beds. Regular skin examinations and early detection are crucial for the successful treatment of skin cancer. Individuals with concerns about their risk of skin cancer should consult with a healthcare professional or dermatologist. Skin malignancies, including non-melanoma and melanoma, have become more common during the past few decades. Presently, each year there are 132,000 cases of melanoma and between 2 and 3 million cases of non-melanoma skin cancer worldwide. According to Skin Cancer Foundation Statistics, one in five Americans will get skin cancer in their lifetime and one in three diagnosed cancers are skin cancers.

Estimates of the number of Americans diagnosed with skin cancer each year range from 2,000 to 8,000. It is estimated that 1.6 billion people of working age (15 years or older), or 28% of the total working-age population, were exposed to UV radiation from the sun while working outdoors in 2020. Nearly 19,000 individuals in 183 countries died from non-melanoma skin cancer in 2020 alone due to of exposure to sunlight outdoors. Of these, the number of men was 65%. These calculations are based on a recent systematic review and meta-analysis published by the World Health Organization (WHO), which showed that occupational exposure to solar UV radiation is associated with a 60% increased risk of non-melanoma skin cancer. This risk estimate was derived from a pooled analysis of 25 case-control studies involving 286,131 individuals from 22 different countries in three different WHO regions. Based on 166 million data points from 763 surveys in 96 countries and territories, including all six WHO regions, which were collected between 1 January 1996 and 31 December 2021, WHO and the ILO estimated the number of individuals exposed to solar UV radiation at work.

There are various methods for identifying cancer in the human body. In a very busy and risky procedure, nearly all surgeons remove tissue from the afflicted organ of the body and submit it for laboratory inspection. In this example, we have five alternatives for skin cancer: Cutaneous carcinoma E_1 , Dermatocarcinoma E_2 , Malignant Skin Neoplasm, and Epidermal Tumor E_5 , and four attributes there:

Asymmetry B_1 : Skin cancer lesions often exhibit irregular shapes, with one half not matching the other.

Irregular Borders B_2 : The edges of a skin cancer lesion may be uneven, with irregular or poorly defined borders.

Varied Colors B_3 : Skin cancer lesions can have a variety of colors, including shades of brown, black, red, white, or blue.

Increased Diameter B_4 : Melanomas, a type of skin cancer, may have a larger diameter (greater than 6 mm) compared to non-cancerous moles.

Evolution or Change B_5 : Changes in size, shape, color, or other characteristics over time may indicate skin cancer.

This example consists of four different types of attributes. Out of the four attributes of skin cancer B_r ($r = 1, 2, 3, 4$) list the above, based on the following 5 candidate's E_r ($r = 1, 2, 3, 4, 5$). The 5 possible skin cancers are to be assessed with IVPFNs $\mathcal{W} = (0.4, 0.1, 0.3, 0.2)$

Table 1: The IVSFN Decision Matrix

	E_1	E_2
B_1	$([0.3, 0.4], [0.3, 0.4], [0.3, 0.5])$	$([0.22, 0.23], [0.31, 0.35], [0.23, 0.25])$
B_2	$([0.2, 0.5], [0.4, 0.5], [0.1, 0.22])$	$([0.32, 0.35], [0.32, 0.33], [0.18, 0.20])$
B_3	$([0.1, 0.3], [0.5, 0.6], [0.2, 0.33])$	$([0.21, 0.25], [0.26, 0.28], [0.17, 0.25])$
B_4	$([0.2, 0.4], [0.2, 0.4], [0.3, 0.44])$	$([0.14, 0.32], [0.28, 0.3], [0.44, 0.45])$
B_5	$([0.2, 0.3], [0.1, 0.3], [0.4, 0.45])$	$([0.42, 0.45], [0.22, 0.35], [0.21, 0.25])$
	E_3	E_4
B_1	$([0.32, 0.35], [0.16, 0.2], [0.31, 0.4])$	$([0.24, 0.25], [0.23, 0.35], [0.18, 0.22])$
B_2	$([0.22, 0.25], [0.15, 0.2], [0.22, 0.25])$	$([0.23, 0.25], [0.21, 0.25], [0.25, 0.3])$
B_3	$([0.21, 0.25], [0.22, 0.25], [0.24, 0.28])$	$([0.22, 0.35], [0.31, 0.37], [0.23, 0.35])$
B_4	$([0.11, 0.15], [0.32, 0.35], [0.26, 0.32])$	$([0.21, 0.32], [0.22, 0.34], [0.22, 0.31])$
B_5	$([0.24, 0.25], [0.21, 0.25], [0.24, 0.33])$	$([0.22, 0.34], [0.17, 0.23], [0.16, 0.22])$

Then, we use the approach developed for skin cancer.

Step 1: According to IVPFNs r_{o_L} ($o = 1, 2, 3, 4, 5, L = 1, 2, 3, 4$), we fuse all IVSFNs r_{o_L} by the IVSFWPDDHM (IVSFWPDDHM) operator to have the IVSFNs B_o ($o = 1, 2, 3, 4, 5$) of the skin cancer B_o . Let $\Gamma = 2$, then the fused results are in Table 3.

Step 2: Calculate the values of $T_{r_\ell}^1, T_{r_\ell}^2, T_{r_\ell}^3$ based on

$$= \begin{pmatrix} 1 & 0.5533 & 0.3259 & 0.2067 \\ 1 & 0.6072 & 0.3928 & 0.2537 \\ 1 & 0.4470 & 0.2785 & 0.1726 \\ 1 & 0.5721 & 0.2970 & 0.1623 \\ 1 & 0.5558 & 0.3893 & 0.2397 \end{pmatrix}$$

Step 3: Calculate the weight of the following values.

Table 2: Calculate the Weights

0.5667	0.3136	0.0528	0.0670
0.5363	0.3257	0.0602	0.0778
0.6153	0.2751	0.0490	0.0607
0.5715	0.3270	0.0485	0.0530
0.5543	0.3081	0.0617	0.0759

Table 3: The Fused Results of the Skin Cancer by the IVSFWPDHM (IVSFWPDDHM) Operator

IVSFWPDHM	
B_1	$([0.0099, 0.9869], [0.9916, 0.9965], [0.0100, 0.0044])$
B_2	$([0.0065, 0.9872], [0.9957, 0.9967], [0.0028, 0.0021])$
B_3	$([0.0039, 0.9800], [0.9951, 0.9959], [0.0064, 0.0047])$
B_4	$([0.0015, 0.9917], [0.9872, 0.9936], [0.0084, 0.0081])$
B_5	$([0.0075, 0.9973], [0.9933, 0.9920], [0.0028, 0.0041])$
IVSWPDDHM	
B_1	$([0.0247, 0.0273], [0.0033, 0.0501], [0.0067, 0.0523])$
B_2	$([0.0199, 0.0153], [0.0017, 0.0068], [0.0016, 0.0278])$
B_3	$([0.0113, 0.0328], [0.0011, 0.0166], [0.0048, 0.0331])$
B_4	$([0.0197, 0.0324], [0.0004, 0.0472], [0.0023, 0.0578])$
B_5	$([0.0237, 0.0113], [0.0018, 0.0231], [0.0006, 0.0385])$

Table 4: The Score Values of Skin Cancer

Alternatives	IVSFWPDHM	IVSFWPDDHM
B_1	0.3453	0.6655
B_2	0.3425	0.6644
B_3	0.3471	0.6669
B_4	0.3430	0.6646
B_5	0.3342	0.6631

Step 4: Table 5 lists the skin cancer in Table 3 order.

Table 5: The Order of the Skin Cancer.

Methods	Order
IVSFWPDHM	$B_3 > B_1 > B_4 > B_2 > B_5$
IVSFWPDDHM	$B_3 > B_1 > B_4 > B_2 > B_5$

Moreover, we find the ideal Malignant Skin Neoplasm is B_3 because B_3 is the highest score value in all of them.

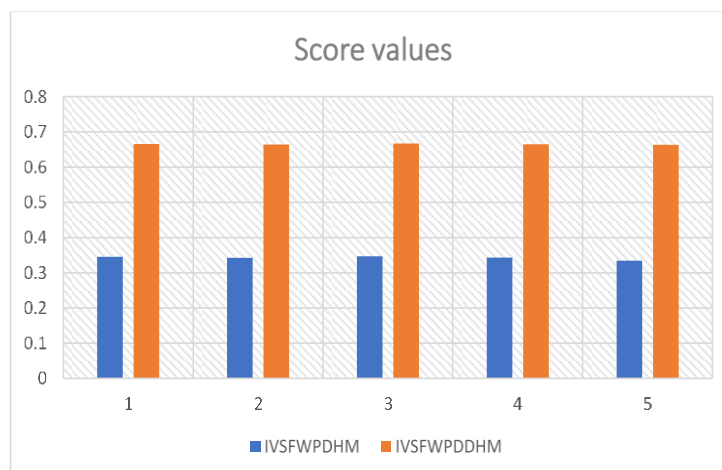


Figure 2: Score Values of Aggregation Operators

4.5 Influence Analysis

The results of the analysis shown in Table 6 illustrate how changes to the parameters of the IVSFWDH and IVSFWDH operators affect ranking outcomes. This investigation underscores the impact of minor parameter adjustments on the ranking of alternatives, demonstrating the operators' sensitivity and adaptability. This kind of analysis guarantees that the proposed model is robust and reliable when dealing with various levels of uncertainty and preferences of DM.

Table 6: The IVSFWDH with Various Parameters

Methods	Score Value					Order
	B_1	B_2	B_3	B_4	B_5	
$X = 1$	0.1609	0.1599	0.197	0.2616	0.2693	$B_3 > B_1 > B_4 > B_2 > B_5$
$X = 2$	0.0817	0.0808	0.0964	0.1547	0.1547	$B_3 > B_1 > B_4 > B_2 > B_5$
$X = 3$	0.146	0.1449	0.1774	0.2424	0.2481	$B_3 > B_1 > B_4 > B_2 > B_5$
$X = 4$	0.2253	0.2256	0.2829	0.3426	0.3578	$B_3 > B_1 > B_4 > B_2 > B_5$

Table 7: The IVSFWDH with Various Parameters

Methods	Score Value					Order
	B_1	B_2	B_3	B_4	B_5	
$X = 1$	0.4816	0.4821	0.4537	0.4795	0.4524	$B_3 > B_1 > B_4 > B_2 > B_5$
$X = 2$	0.4463	0.4532	0.4169	0.4428	0.4129	$B_3 > B_1 > B_4 > B_2 > B_5$
$X = 3$	0.5812	0.5599	0.5598	0.5852	0.5642	$B_3 > B_1 > B_4 > B_2 > B_5$
$X = 4$	0.3884	0.4038	0.3573	0.3832	0.3484	$B_3 > B_1 > B_4 > B_2 > B_5$

The result is that different parameters have the same B_3 is the best value in all of them.

4.6 Sensitivity Analysis

An integral component of any MCDM problem is the weight vector. To determine the robustness of our ranking results and to confirm the validity of the selected DM method, we perform a post-sensitivity analysis in this subsection. Therefore, we alter the original weight vector per two schemes: (1) raising the weight of each attribute by 0.05, and (2) lowering the weight of each attribute by 0.05. To maintain the normalization of the modified weight vector in both cases, the weights of the remaining criteria are modified using the equation (7) proposed by Xu and Yager (2008).

$$\omega_c = \left(\frac{1-\omega_c}{1-\omega_*} \right) \omega_* \quad (7)$$

Where ω_* represents the changed weight, ω_c is the resultant attribute weight, and ω_c is the attribute's associated weight.

Table 8-10 shows the computed weight values for the expected strategy. These weights form the basis for assessing the intended MCDM approach's sensitivity. Ten different sensitivity analysis scenarios were carried out in order to evaluate the DM results' robustness. Table 8-10 lists the associated score values and the alternative rankings that result for each scenario.

According to the data presented in Table 10, each scenario entails a ± 0.05 perturbation of a single attribute weight while keeping the other weights constant. The analysis shows that the alternatives' ranking is extremely stable across all ten tested scenarios, with a consistent.

Table 8: The Weight combination of DHM and DHM

Combination #	Weight 1	Weight 2	Weight 3	Weight 4
1	0.3500	0.3500	0.1000	0.2000
2	0.3500	0.3500	0.2000	0.1000
3	0.3500	0.1000	0.3500	0.2000
4	0.3500	0.1000	0.2000	0.3500
5	0.3500	0.2000	0.3500	0.1000
6	0.3500	0.2000	0.1000	0.3500
7	0.1000	0.3500	0.3500	0.2000
8	0.1000	0.3500	0.2000	0.3500
9	0.1000	0.2000	0.3500	0.3500
10	0.2000	0.3500	0.3500	0.1000
11	0.2000	0.3500	0.1000	0.3500
12	0.2000	0.1000	0.3500	0.3500

Table 9: The Result of Weights DWHM

	Score Value					Order
	B_1	B_2	B_3	B_4	B_5	
1	0.3453	0.3425	0.3471	0.3430	0.3342	$B_3 > B_1 > B_5 > B_4 > B_2$
2	0.3455	0.3421	0.3502	0.3438	0.3342	$B_3 > B_1 > B_5 > B_4 > B_2$
3	0.3478	0.3443	0.3466	0.3430	0.3330	$B_3 > B_1 > B_5 > B_4 > B_2$
4	0.3469	0.3444	0.3449	0.3428	0.3329	$B_3 > B_1 > B_5 > B_4 > B_2$
5	0.3466	0.3427	0.3497	0.3436	0.3336	$B_3 > B_1 > B_5 > B_4 > B_2$
6	0.3456	0.3431	0.3448	0.3424	0.3336	$B_3 > B_1 > B_5 > B_4 > B_2$
7	0.3463	0.3401	0.3443	0.3416	0.3328	$B_3 > B_1 > B_5 > B_4 > B_2$
8	0.3460	0.3402	0.3424	0.3407	0.3329	$B_3 > B_1 > B_5 > B_4 > B_2$
9	0.3462	0.3408	0.3422	0.3406	0.3323	$B_3 > B_1 > B_5 > B_4 > B_2$
10	0.3458	0.3408	0.3484	0.3429	0.3334	$B_3 > B_1 > B_5 > B_4 > B_2$
11	0.3452	0.3413	0.3435	0.3413	0.3335	$B_3 > B_1 > B_5 > B_4 > B_2$
12	0.3466	0.3433	0.3433	0.3415	0.3322	$B_3 > B_1 > B_5 > B_4 > B_2$

Table 10: The Result of Weights DDHM

	Score Value					Order
	B_1	B_2	B_3	B_4	B_5	
1	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
2	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
3	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
4	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
5	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
6	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
7	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
8	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
9	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
10	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
11	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
12	0.6661	0.6635	0.6665	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$

4.7 Comparative Analysis

We compare the IVSFWPDHM and IVSFWPDDHM operators with the Sarfraz et al. (2022) gave the new concept of prioritized AOs based on the AA TN and TM using the applications in group decision making. Mandal and Seikh (2023) gave the concept of Dombi AOs on the basis of IVSF. Wei and Wei (2018) prioritized weighted AOs on MADM. The results are listed in Table 11.

Table 11: The Arrangement of Skin Cancer

Methods	Score Value					Order
	B_1	B_2	B_3	B_4	B_5	
IVSFWPDHM	0.3453	0.3425	0.3471	0.3430	0.3342	$B_3 > B_1 > B_4 > B_2 > B_5$
IVSFWPDDHM	0.6655	0.6644	0.6669	0.6646	0.6631	$B_3 > B_1 > B_4 > B_2 > B_5$
Sarfraz et al. (2022)	0.3937	0.3372	0.4643	0.413	0.41302	$B_3 > B_1 > B_4 > B_2 > B_5$
Sarfraz et al. (2022)	0.3231	0.1693	0.3244	0.2935	0.2064	$B_3 > B_1 > B_4 > B_2 > B_5$
Mandal and Seikh (2023)	0.9767	0.9388	0.9898	0.9411	0.9017	$B_3 > B_1 > B_4 > B_2 > B_5$
Mandal and Seikh (2023)	0.3101	0.2721	0.3231	0.2743	0.2349	$B_3 > B_1 > B_4 > B_2 > B_5$
Wei and Wei (2018)	0.3394	0.2599	0.3893	0.3513	0.2247	$B_3 > B_1 > B_4 > B_2 > B_5$
Wei and Wei (2018)	0.2874	0.1662	0.3694	0.2747	0.1073	$B_3 > B_1 > B_4 > B_2 > B_5$

From the above example, IVSFWPDDHM gives the same result in different operators, so we can say that the best alternative is B_3 , but IVSFWPDHM gives the same result in different operators, so we can describe the best alternative. However, the current technique still uses the same data. The results of our suggested IVSFWPDHM and IVSFWPDDHM

operators were identical.

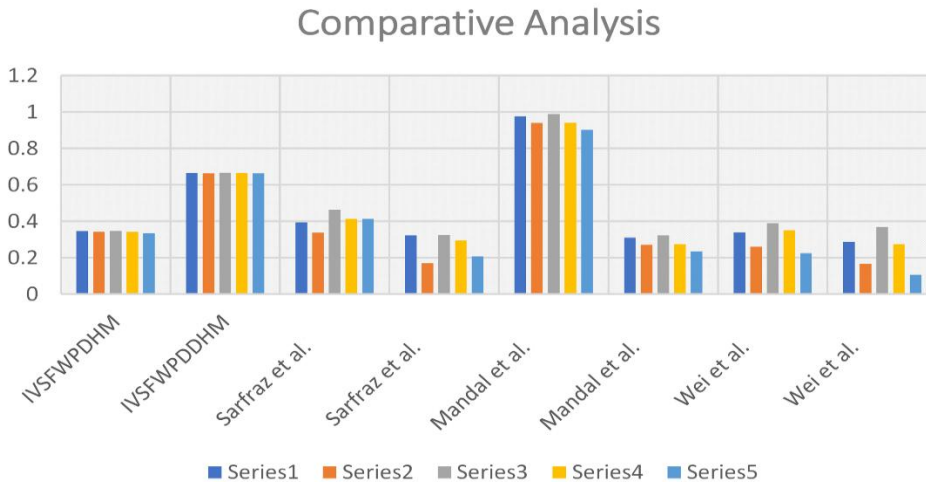


Figure 3: Graphical Representation of Ranking Outcomes of Different Approaches

4.8 Advantages of the Proposed Study

- The developed operators IVSFPDHM and IVSFWPDDHM are capable of managing interval-valued and SF information, leading to enhanced decision accuracy in uncertain situations.
- With the prioritized structure, DMs can assign varying levels of importance to attributes, mirroring actual medical priorities in the real world.
- Flexibility in aggregation is enhanced by the Dombi operations, allowing for a smooth adjustment between logical conjunction and disjunction.
- The reliability of the diagnosis is enhanced by the HM approach, which captures interrelationships among medical attributes.
- The suggested approach is capable of adapting to the complexities and uncertainties typical of medical DM environments.
- The comparative analysis demonstrates that the proposed operators yield results that align with previous studies while providing enhanced stability and precision.

5. Conclusions

The present study concentrated on problems of aggregating IVSF information that involve a prioritization relationship among the criteria. Two new operators, IVSFWPDHM and IVSFWPDDHM, were proposed based on the concept of PAOs. These operators have a significant advantage in that they can take into account the priority structure among decision criteria, which makes them more applicable to complex decision environments. The proposed operators were subjected to a thorough analysis of their mathematical properties. Furthermore, a MADM approach was developed under the IVSF environment using these operators. Finally, a practical example was presented to demonstrate the effectiveness and applicability of the proposed method. By incorporating priority relationships between DMs and criteria, the flexibility and practical relevance of the proposed MADM framework are enhanced.

5.1 Limitation

- The study focuses exclusively on medical diagnosis problems and does not investigate other important decision-making domains, such as engineering or environmental management.
- The model relies on pre-established weight information, which might not completely reflect the subjectivity of medical experts in real-world scenarios.
- The suggested operators could encounter computational difficulties when used on data that is either very large in scale or highly complex.

5.2 Future Study

- The fact that this paper's findings can be applied to complex spherical fuzzy, T-spherical fuzzy, complicated T-spherical fuzzy, and hesitant fuzzy environments is meaningless.

- Enhance robustness by creating hybrid methods that integrate these operators with statistical or optimization techniques.

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