

Assessing the Relationship Between Mathematical Rigor and Metacognitive Thinking Among Secondary School Teachers

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ABSTRACT

The present study examined the association between secondary mathematics teachers' perceptions in terms of rigor and metacognitive thinking, encompassing two cognitive dimensions that influence effective instruction. Teachers' perceptions regarding mathematical rigor ranged from narrow and complex calculations and algorithmic accuracy to broader, more integrated perspectives that include procedural fluency, conceptual understanding, strategic competence, adaptive reasoning, and productive mathematical dispositions. This research explored the interaction between secondary mathematics teachers' perceptions of mathematical rigor and their levels of metacognitive thinking. The findings indicate that teachers demonstrated moderate proficiency in instructional practices related to both mathematical rigor and metacognitive thinking. Moreover, a statistically significant moderate positive correlation was identified, suggesting that higher metacognitive awareness is associated with stronger enactment of rigorous mathematical teaching practices. The study recommends implementing comprehensive professional development programs and utilizing the study's instruments as diagnostic tools to strengthen teachers' competence in both mathematical rigor and metacognitive thinking.

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1. Introduction

This era is defined by profound and rapid change, where knowledge plays the role of both product and driver of progress (Zou et al., 2025). In the present complex environment, the status and role of education have evolved and shifted from a simple knowledge transmission to a critical mission of empowerment. It is important that educational institutes equip students with cognitive tools so they can thrive in a competitive global environment, contribute to society in a meaningful manner, and navigate ambiguity (Castro Laszlo & Laszlo, 2002). It necessitates an alteration toward comprehensive education that improves what is learned by students along with their thinking processes (Vettriselvan, 2025).

At the base of the evolution of education is metacognition, the regulation and awareness of a person's own thinking processes. Moving beyond algorithmic problem solving, the metacognitive style of teaching provides opportunities to learners so they can critically examine the way they think (Torregrosa & Albarracín, 2025). It is based on negotiating ideas, seeking improvement on a continuous basis, evaluating strategies, and making distinctions. It leads to an efficient as well as economically oriented mindset toward the production of abundant ideas, innovative ideas, and high-quality ideas (Liu & Zhao, 2022). Moreover, it is based on awareness regarding the effective domain to understand their own intentions, motivations and feelings behind their thought process (Popa, 2022).

In this process, teachers are one of the key catalysts. Through modeling of intentionally engaging and reflective thinking with students, and their own ideas, educators are able to stimulate learners thought about their thinking (ElSayary, 2025). This practice is based on achieving deep learning and building self-confidence that in turn improves students' enjoyment and abilities in the learning process. With the help of guided metacognition, students become proficient at designing effective procedures, contextualizing knowledge, and integrating disparate information for decision making and problem solving. They evolve into application-oriented, initiative-taking learners who become skilled in understanding and collaboration within an engaging and active learning environment (Fahrni et al., 2025).

In the metacognitive framework, mathematics of school has very unique as well as powerful position. This course

and discipline are designed to develop logical reasoning which is the primary goal of education (Purwaningsih et al., 2025). Mathematics is considered an integrated system that is built on freedom, rigor, and consistency without contradiction. The nature of this discipline requires logical thinking that encourages the mind to divide problems into different parts, understand the way they are connected, and look for solutions to these problems in a disciplined and creative way. The process guided by the vital and precise logic of mathematics is one of the very important steps in seeking solutions (Fitriani & Widjajanti, 2024).

Most of the time, mathematics is considered intimidating, dry, and complex, reactions stemming from psychological and cultural biases rather than the inherent qualities of the subject. Whereas mathematics is one of the superior tools to understand the relational and quantitative fabric of education and the world. Because of the precision of mathematics, it is indispensable for scientific analysis, technological innovation, and problem solving (Jean-Pierre & Jean-Pierre, 2026). With the purpose of handling negative perceptions, instruction must be implemented by teachers that is facilitating, motivating, and enriching. As a result of this approach, students' aversion and anxiety will be reduced, their interest will rise, and they will develop a positive attitude (Atoyebi et al., 2023).

Effective teaching of mathematics that bridges metacognition and rigor includes creating a safe space for analysis of errors that allows students to handle mistakes within the concepts of mathematics, stimulating doubts regarding science that encourage students to question and re-examine ideas in a respectful way (Tañola & Lomibao, 2024), treat logical Fallacies in the form of opportunities of learners, that addresses misconceptions through examples and more meaningful learning, and fostering intellectual partnership that clarifies role of learners that involves pushing students to short list different ideas and assumptions (Varveris et al., 2023).

In different institutions, mathematics is taught using the lens of metacognition and rigor. The classroom becomes a community where intellectual risk-taking is encouraged. Students value mathematical rigor in the form of cognitive safeguards in order to prevent different errors (Boaler & Dieckmann, 2026). Students become more receptive to formative feedback and develop an appreciation for the profound connection of mathematics to the interpretation, understanding, and problem analysis of real-world contexts. As a result of this experience, fear, anxiety, and preconceptions are reshaped into competence and confidence. Therefore, students do not perceive mathematics as an abstract and dry subject (Toker, 2022). In fact, they perceive it as a relevant and powerful tool that can enrich students' experiences and fuel scientific development in the region. Ultimately, by focusing on metacognitive thinking, teachers do not only provide support to mathematics teaching, they teach students the way to think confidently and clearly, and the skill level of students will be enhanced.

Despite the recognized importance of metacognition, the relationship between the construction of metacognition and rigor is explored insufficiently. Whereas, past studies have explored metacognitive thinking separately (Katyal & Fleming, 2024). Therefore, very little knowledge exists regarding the way teachers perceived about rigor and its impact on metacognitive awareness (McKendree & Washburn, 2021). Therefore, the main purpose of this study is to address this gap by analyzing the relationship among secondary mathematics teachers in terms of their perceptions regarding mathematical rigor and their level of metacognitive thinking. The basic research problem is to determine the nature of the relationship between teachers' perceptions regarding mathematical rigor at the secondary school level and their level of metacognitive thinking. On the basis of this stated research problem, this study tends to explore to what extent secondary school mathematics teachers demonstrate a developed understanding and application of mathematical rigor in their profession, to what extent metacognitive thinking is exhibited by secondary school teachers in terms of instructional reflection and planning, and whether there is any correlational significance between metacognitive thinking and rigor among secondary school mathematics teachers.

2. Literature Review

2.1 Mathematical Rigor

Mathematical rigor reflects the foundational pillar regarding advanced thought of mathematics and instructions that are extended beyond correction of procedure to develop a coherent, logical and deep engagement with the content of mathematics (Baez et al., 2025). Scholars mentioned that essence of mathematics rigor lies in the volume of resources and work along with the coherence of solution, control of relationships and precision of procedure. Mathematics rigor is characterized by avoidance of error in disciplined way, pursuit of efficient and economical solutions, and conscious awareness of methodological choices (Gordon, 2021). Ultimately, rigor works as a superior principle in transcending any standard, idea and teaching.

Studies revealed that mathematical rigor has roots in the rejection of insistence and contradiction on the level of understanding the methods that are used to analyze context of mathematics by different logical construction (Hamami, 2014). As a result of this process, ambiguity is mitigated through usage of mathematical ideas, abandoning, modifying or

adopting them through scientific flexibility and curiosity. This takes place within environment of a classroom that is based on critical review, fostering a deep and shared responsibility for interconnections and concepts. The main focus is to control the confusion, sustain mathematics as enriching and convincing discipline and achieve stability in mathematics discipline that extend the level of awareness among learners (Siregar, 2025). Scholars have mentioned two types of rigors, namely instructional rigor and content rigor. The instructional rigor deals with the conscious creation of cognitive environment through engagement and interaction with acceptance of complexity and high-quality tasks. Whereas context rigor deals with the learner conceptual mastery, support for deep understanding and depth of conceptual skills (Marzano & Toth, 2014).

This philosophy is operationalized by the content unification with different standards of practices of mathematics making the functional meaning of academic rigor specific and apparent. In turn, teachers are guided by it to develop practices that empower learners with in-depth understanding. Scholars have explained the concept of rigor as foundational experience playing the role of starting point for exploration of mathematics, characterization of different tasks by level of challenge and difficulty, strict commitment to imposition of content and employing difference from the level of hypothesis to proof (Grotzer & Mittlefehldt, 2011).

Academic rigor work as the indicator to fulfill the demands and needs of learning that is characterized by cognitive passion and curiosity. It is achieved by challenging different tasks that are needed for high level of perception. Therefore, a person is able to develop mathematical values, beliefs and skills (Wright, 2021). It is moved beyond inference, justification, precision and communication to actively stimulate, direct and invest the learner's response that guides them to get correct answers. Past studies have focused on a number of different components leading to dimensions including task content, task implementation, post task discussion and task expectations (Skehan, 2016). The higher task content offers multiple solution strategies and entry points, foster mathematical reasoning and communication, allowing diverse treatment on the basis of experience of students and utilizing multiple opportunities and representations for connection. On the other hand, effective task implementation needs to move beyond procedural and conventional methods. It demands strategic teacher support, collaborative thinking, sufficient and focused information, and flexibility to guide the students and teachers (Lakkala et al., 2021). Whereas post task discussion is based on completion of tasks given allowing students to form connections, explain and analyze ideas, and compare and examine strategic solutions. In the end, the task expectations focus on successful implementation of high-level tasks.

Scholars developed an evaluative instrument to measure academic rigor as criteria to assess elementary mathematic programs. Results shed that there is variation in quality and rigor of mathematics learning opportunities across all educational instruments (Gallagher et al., 2025). This study highlights rigor as important component to enhance flexibility in system and improve professional development of teachers (Boston & Wolf, 2006). The research by Johnson (2010) used mixed methodology and mentioned rigor as important value factor for effective teaching.

Likewise, the study by Adnan (2024) evaluated pre-service teachers of mathematics in terms of integration of their ability and technology. Results revealed that participants successfully deleted and created high level implementation and deductive reasoning showing strong justifying conclusion and mathematical thinking. Whereas scholars with the vase study showed in their study that teacher's attitude is positively affected by learning communities (McDonough, 2013). Scholars explored perception of undergraduate students and reported importance of active learning approach in their study to promote meaningful academic challenges, appropriate expectations and high-level thinking (Gasmi & Al Nadabi, 2023). The study by Banner (2016) used two matrices where one focused on assessment of planning rigor and other on assessment of implementation rigor. Results showed improvement in leadership and teacher understanding related rigor.

Scholars conducted study and reported complex relationship between mandated curricula and preferences of teachers in terms of teachings of Algebra (Diletti, 2017). On the other hand, the study of Buser (2018) reported positive impact of professional development programs that are developed through academic rigor. Results also concluded that training should be given to teachers to pose targeted and deep questions related to mathematics when they are teaching students in the classroom environment.

2.2 Studies on Metacognitive Thinking

Metacognition also known as "thinking about thinking" is the base of self-regulated learning. For a student to achieve difficult and complex tasks, the students need to understand their own procedures, concepts and styles (Marantika, 2021). This closes in the knowledge of metacognition as awareness of one's strategic and tactical knowledge and ability to monitor perception of one's own. Basically, it is the knowledge of individuals of their capacity and knowledge that enables them to reorganize and change their thoughts as per the demands of learning (Anthonysamy et al., 2025). There are different components of metacognition concepts namely metacognitive knowledge and metacognitive regulation. Studies have mentioned three types of metacognitive knowledge namely conditional knowledge, procedural knowledge and declarative

knowledge. On the other hand, metacognitive regulation is based on different processes namely evaluation, monitoring and planning (Shekh-Abed, 2025).

For the teachers of mathematics, this concept translates to follow the procedure to improve their own awareness and guide as per the context of teaching. It is based on critical examination of learner acceptance, classroom leadership method and task design (Bognar et al., 2025). The main purpose is to support the ability of learners to handle their own learning, evaluating it, monitoring it, controlling and thinking it. As a result of this self-regulation, a person is led to accept the rigor (Faza & Lestari, 2025), as a convincing pathway to produce correct solutions through responsive awareness, meaningful understanding and deep awareness. The harmony between learner, teacher and content should be aligned that is advocated for the responsibility of learners to construct their own knowledge. Learning is developed through renewal of knowledge procedurally and conceptually, mathematical thinking and problem solving. It is enriched by idea generation, negotiation of solutions, argument evaluation and intellectual flexibility that collectively achieve positive beliefs regarding mathematics, deep understanding and mathematical confidence (Nilimaa, 2023).

Past studies also proposed seven step models to teach metacognition in Mathematics that start with activation of learners' prior knowledge of mathematics that is followed to provide explicit instruction and modeling effective performance of metacognition. It later focuses on consolidating key ideas of mathematics before moving to the practices that are guided to support application of these strategies (De Almeida & de Castro, 2023).

Past studies assessed the level to which teachers of mathematics have the competence of metacognition by using questionnaires. The results showed that there is moderate level of control, monitoring, evaluation and planning among the teachers. Results also reported that there is significant difference on the basis of gender to have skills of metacognition (Vanags & Pestovs, 2019). Scholars also demonstrated the effectiveness of metacognitive strategy to enhance motivation and generative thinking among students (Li et al., 2025).

Scholars have listed essential skills of metacognition in order to teach mathematics. They examined the level to which these skills are practiced by teachers. Results also showed that high level of metacognitive practices have higher differences in favor of more teachers (Güner & Erbay, 2021). Scholars conducted an observational study to document metacognitive skills being used by the teachers of mathematics and students when they are using problem solving activities (Ambaryani & Putranta, 2022). The results of the study identified different patterns that are clear. The planning phase involves interpretation of procedural steps, monitoring and maintaining procedural order, and evaluation of Hao et al. (2025). Scholars also examined the impact of effectiveness of training programs that were designed to help the teachers at middle school mathematics who use metacognitive strategies. In the experimental group, teachers demonstrated high level of usage of these strategies. The students performed very well in the tasks of problem-solving (Alfayez et al., 2022). Results by Yorulmaz et al. (2021) also reported that primary teachers of mathematics have high level of awareness regarding metacognition. They also possess moderate confidence in solving a problem and moderate correlation among variables. Likewise, scholars developed quantitative study and showed that teachers at primary school mainly rely on cognitive skills. On the other hand, metacognitive strategies are extensively used by teachers at secondary school (Ozturk, 2021). Furthermore, the results by Wafubwa and Csikos (2021) in context of Turkey also found that using formative assessment techniques have positive effect on metacognitive skills.

The literature review is based on studies from last three decades related to metacognitive thinking and mathematical rigor. All these studies were included from around the globe. The educational stages, samples, tools and methodologies were targeted in these studies and were highly varied. The topics that were explored were related to academic rigor, structure and philosophy. Moreover, their relationship was assessed through professional communities, and educational quality along with their connection to teacher role and technology integration.

Table 1(a): Summary of Previous Studies' Focus and Context

Study Author (Year)	Location	Focus Area	Key Findings
Boston and Wolf (2006)	California, USA	Rigor as an Evaluation Standard	Found variation in rigor and learning opportunities between schools.
Johnson (2010)	Arizona, USA	Professional Communities & Rigor	PLCs effective in increasing rigor and improving outcomes.
Akcay and Boston (2018)	Pennsylvania, USA	Tech Integration & Rigor	Pre-service teachers created high-cognitive-demand tech tasks.
McDonough (2013)	Texas, USA	PLCs, Morale & Rigor	PLCs positively affected teacher morale and rigorous teaching.
Draeger et al. (2015)	New York, USA	Student Perceptions of Rigor	Students linked rigor to challenge, interest, and active learning.
Banner (2016)	Connecticut, USA	Frameworks for Rigor	Matrices for implementation and planning improved teacher rigor.
Diletti (2017)	New York, USA	Math Transformations & Rigor	Found weak teacher use of transformations for achieving rigor.

Table 1(b): Summary of Previous Studies' Focus and Context

Study Author (Year)	Location	Focus Area	Key Findings
Buser (2018)	Pittsburgh, USA	Training for Rigorous Practices	Training in rigorous questioning improved student reasoning.
Yorulmaz et al. (2021)	Turkey	Metacognitive Awareness	Pre-service teachers had high awareness, medium problem-solving belief.
Peñarroja (2021)	Kenya	Formative Assessment & Metacognition	Formative assessment predicted metacognitive regulation skills.
Ozturk (2021)	Turkey	Cognitive vs. Metacognitive Skills	Secondary teachers used more metacognitive skills than primary.

3. Research Methodology

This study employed quantitative research for the investigation of research questions and hypothesis. The basic purpose was to examine the relationship between metacognitive thinking and mathematical rigor among secondary school teachers of mathematics.

The population of study is based on male teachers of Mathematics that were teaching in different public schools in AlAhsa, Saudi Arabia. For this purpose, present study employed purposive sampling. Data was gathered from 122 teachers in 34 different schools. This study used questionnaires as tool for data collection. The items of Mathematical Rigor scale were adapted Buser (2018), Diletti (2017), Banner (2016), Draeger et al. (2015). For measurement of metacognitive thinking, scale was developed from the studies of Ozturk (2021), Wafubwa and Csikos (2021), Yorulmaz et al. (2021). On the basis of subsequent modifications and expert reviews, the final instruments were comprised of 17 items and 36 items respectively.

The scales were initially presented to a panel of arbitrators consisting of faculty members specializing in mathematics teaching methods, psychology, and measurement and evaluation. Based on their feedback and suggestions, the researcher reviewed the item formulation and considered some of the proposed alternatives. Consequently, modifications were made to the scales.

To ensure psychometric soundness, the scale was piloted on a sample of 29 mathematics teachers who were not part of the main study cohort. The construct validity of the scale was confirmed through various indicators. First, its validity was established through internal consistency checks. Strong, statistically significant Pearson correlation coefficients were observed between each individual item and its corresponding dimension total as shown in Table 2. These results confirm that all items effectively measure their intended construct and that all dimensions contribute significantly to the overall scale.

Table 2: Item-Total Correlation Coefficients for the Mathematical Rigor Scale (MRS) and Metacognitive Thinking Scale (MTS)

Item	Correlation	Item	Correlation	Item	Correlation	Item	Correlation
TC1	0.684	TI3	0.681	DT3	0.958		
TC2	0.772	TI4	0.743	DT4	0.969		
TC3	0.775	DT1	0.675	DT5	0.957		
TI1	0.778	DT2	0.829	TE1	0.956		
TI2	0.635	DT3	0.741	TE2	0.970		
Item	Correlation	Item	Correlation	Item	Correlation	Item	Correlation
KC1	0.551	KC10	0.66	RC6	0.505	PC3	0.744
KC2	0.677	KC11	0.682	RC7	0.653	PC4	0.695
KC3	0.735	KC12	0.738	RC8	0.647	PC5	0.751
KC4	0.739	KC13	0.717	RC9	0.666	PC6	0.693
KC5	0.565	RC1	0.679	RC10	0.617	PC7	0.799
KC6	0.604	RC2	0.554	RC11	0.653	PC8	0.725
KC7	0.71	RC3	0.579	RC12	0.654	PC9	0.837
KC8	0.609	RC4	0.644	PC1	0.76	PC10	0.777
KC9	0.759	RC5	0.605	PC2	0.776	PC11	0.682

Second, Key indicators and coefficients demonstrate whether the scales are valid. Macdonald's Omega and composite reliability (CR) are commonly used to assess the scales' reliability. According to the findings presented in Table 3, both Macdonald's Omega and CR fall within the range of 0.795–0.863 and 0.763–0.818, respectively for Mathematical Rigor Scale (MRS), which align with the suggested values (> 70%), and for Metacognitive Thinking Scale (MTS) within the range

of 0.948-0.963 and 0.938-0.966, signifying a level of consistency for the scale's measurements.

Furthermore, the Average Variance Extracted (AVE) values for Mathematical Rigor Scale (MRS) range from 56% to 63%, and for Metacognitive Thinking Scale (MTS) range from 52% to 56% which is greater than 50%, indicating a level of variance explained by the constructs being measured. In terms of validity coefficients (the root of the AVE or convergent validity), it is important that these coefficients exceed the intercorrelations between the latent variables or factors. Comparing the last column with the minimum loading factor value and observing that it significantly surpasses it indicates the reliability and validity of the scale.

Table 3: Indicators and Coefficients of Construct Validity for both Scales.

Dimensions of MRS	Items	Loading Factor	Macdonalds Omega	CR	AVE	$\sqrt{AVE}$
Task Content (TC)	4	0.71-0.88	0.845	0.802	0.613	0.783
Task Implementation (TI)	4	0.63-0.79	0.863	0.818	0.625	0.791
Discussions About the Task (DT)	5	0.58-0.70	0.795	0.763	0.562	0.749
Teacher's Expectations of Task Performance (TE)	4	0.49-0.81	0.844	0.811	0.617	0.785
Dimensions of MTS	Items	Loading Factor	Macdonalds Omega	CR	AVE	$\sqrt{AVE}$
Knowledge of Cognition (KC)	13	0.48-0.92	0.951	0.938	0.527	0.726
Regulation of Cognition (RC)	12	0.72-0.88	0.948	0.954	0.559	0.748
Processing of Cognition (PC)	11	0.76-0.91	0.963	0.966	0.518	0.719

Collectively, the results presented in Tables 2 and 3 demonstrate that the Mathematical Rigor Scale possesses strong validity and high reliability, providing confidence in the accuracy and consistency of the data obtained through its application.

3.1 Instrument Design and Scoring

Significant care was taken during the item-drafting phase to ensure linguistic accuracy, clarity of meaning, operational definability, and the avoidance of ambiguity and redundancy. The items were phrased in a negative form to mitigate acquiescence bias and encourage more thoughtful responses.

Participant responses were measured using a five-point Likert scale (ranging from 1 = Strongly Disagree to 5 = Strongly Agree). The total scores for each scale, for both individual responses and sample means, fall within the interpretative ranges detailed in Table 4. This framework allows for a clear judgment of the level of each variable.

Table 4: Interpretation Framework for Scale Scores

Judgment	Likert Value	Math. Rigor (15 Items)	Meta. Thinking (36 Items)	Mean Per Item
Very High	5	70 – 85	150– 180	4.20 – 5.00
High	4	55 – 70	120 – 150	3.40 – 4.20
Moderate	3	40 – 55	90 – 120	2.60 – 3.40
Low	2	25 – 40	60 – 90	1.80 – 2.60
Very Low	1	10 – 25	30 – 60	1.00 – 1.80

3.2 Procedures and Data Collection

The field application was conducted during the fifth and sixth weeks of the first semester of the 2025/2026 academic year. To ensure a smooth and standardized administration process, the researcher coordinated with the Education Department in Al-Ahsa.

An introductory meeting was held via the Blackboard platform to explain the study's objectives, ensure confidentiality, and provide instructions for completing the instruments. Following this, the digital survey links for both scales were distributed to all 71 teachers in the sample. The data collection process yielded a 100% response rate with no missing data, resulting in a complete dataset from the entire purposive sample for statistical analysis.

4. Results

The findings of the study, which address the research questions regarding the levels of mathematical rigor and metacognitive thinking among secondary school teachers and the relationship between them, are presented below.

4.1 Levels of Mathematical Rigor and Metacognitive Thinking

To answer the first and second research questions, descriptive statistics were calculated for both variables across the entire sample (N=71). The results, including the means, standard deviations, and the corresponding interpretative level, are summarized in Table 5.

Table 5: Descriptive Statistics and Level of Mathematical Rigor and Metacognitive Thinking

Variable	Mean	Std. Deviation	Std. Error	Level
Mathematical Rigor	50.35	15.72	1.87	Moderate
Metacognitive Thinking	108.52	30.12	3.58	Moderate

As illustrated in Table 5, the mean scores for both mathematical rigor ($M = 50.35$) and metacognitive thinking ($M = 108.52$) fall within the "Moderate" range according to the pre-established interpretative framework (Table 9). This indicates that, on average, the participating teachers demonstrate a moderate level of competency in both implementing rigorous mathematical teaching practices and applying metacognitive strategies.

4.2 Correlation between Mathematical Rigor and Metacognitive Thinking

The third research question aimed to determine if a statistically significant correlation exists between the two variables. A Pearson product-moment correlation was computed, and the result is presented in Table 6.

Table 6: Pearson Correlation

Pearson Correlation Coefficient	Sig. (2-Tailed)	Correlation Level
0.510	0.007	Moderate Positive

** $p < 0.01$

The analysis revealed a statistically significant, moderate positive correlation between mathematical rigor and metacognitive thinking, $r^*(69) = 0.51$, $p^* = 0.007$. This indicates that as teachers' propensity for metacognitive thinking increases, so does their implementation of mathematically rigorous teaching practices.

5. Discussion

In this section results of the present study will be interpreted and will be linked to past studies. The results of the study revealed that metacognitive thinking and mathematical rigor were perceived at moderate level. It shows foundational competency and consistency among teaching cohort. This can be considered as positive indication teachers are moving as per procedural-based instructions and traditional instructions. This moderate level reflects conscious effort with purpose to ensure that learners achieve a detailed understanding of their subject and balancing conceptual comprehension with fluency. Such teaching practices are metacognitive by nature. It is based on guidance of teachers regarding collaborative thinking, critiquing connections, negotiating mathematical ideas and deeper analysis. The main goal of rigorous approach is to enable teachers to diagnose and reflect on the root cause of errors. Because of this awareness learners are able to find out logical fallacies, and conceptual deficiencies that are important for mastering mathematical tasks. This improves the environment of a classroom where value is given to students who are more receptive to feedback.

The result showing positive correlation (Moderate) between metacognitive thinking and mathematical rigor is an important finding. This finding shows the important theoretical link that is used to underpin the relationship between rigorous mathematical teaching and metacognitive awareness. It is possible to explain this relationship by the fact that necessary framework is provided by metacognitive skills for the implementation of rigor. Teachers who have higher metacognitive awareness are equipped better to understand cognitive shortcomings of the learner. They understand that discussion of mathematics is important for solidifying understanding, correction and reviewing. From this perspective, teachers are able to convey their lecture skillfully through fluency and understanding that encourages the engagement and appreciation of students in terms of mathematics.

The positive moderate correlation also suggests that there is symbiotic relationship where metacognitive thinking and mathematical rigor are reinforced mutually. Both these factors operate in a dynamic cycle. The metacognitive planning of teachers anticipates the difficulties of students, designing tasks that need justification and planning probing questions for rigorous environment of classroom. On the other hand, their engagement in rigorous practices hones metacognitive skills of the teacher, inherently demands and evaluate multiple solution strategies. They must monitor understanding of students on constant basis, evaluate the effectiveness of selected task of mathematics and regulate their own instructional approach. These results portray metacognition not only as secondary option but as basic pedagogical skill essential to achieve the precision and depth that is required by mathematical rigor.

The results that moderate level of metacognitive thinking is similar to the results of Buser (2018) who also demonstrated moderate possession of metacognitive competencies among teachers of mathematics. Whereas this study is opposite to the results of Yorulmaz et al. (2021) who integrated higher level of metacognitive awareness and skills. These differences in terms of specific instruments, professional development opportunities and cultural contexts. Moreover, the correlational findings have strong relationship with contemporary research. This findings is consistent with the results of

Wafubwa and Csikos (2021), who reported strategies of formative assessments to have positive effect on the strategic learning behavior of students. This also aligned with the results of Yorulmaz et al. (2021), who reported moderate correlation between the teaching beliefs and metacognitive awareness. On the other hand, these results contradicts the findings of Diletti (2017), who mentioned weaknesses to employ transformation of mathematics to achieve cognitive rigor.

6. Limitations and Future Directions

This section will discuss limitations and directions for the future authors of the study. The result of the study focuses on the thinking skills and rigor. This study ignores the integrated approach. Therefore, the training designed for the trainers must develop prototypes of the training of the teachers that define the complete mechanism. Moreover, the correlation shows moderate strength. It shows there are other factors that has the capability to affect the mathematical rigor. Therefore, other factors must be considered in future studies to enhance the level of rigor. These contents can be available resources, curriculum mandates, institutional constraints and teachers' skill. For the analysis, the studies in future should focus on multivariate analysis.

Additionally, the finding focused on quantitative approach and ignored qualitative nature of the study. The result show moderate findings of the proposed questions. Therefore, the study in future should focus on qualitative approach so the results of these assumptions may improve. In other words, the upcoming studies should focus on conducting observational and qualitative studies, investigate the effectiveness and role of professional learning communities, assess and develop impact of targeted training programs, and explore potential variation to demonstrate metacognitive thinking and academic rigor. Moreover, the data of the present study was collected by using purposive sampling. Basically, purposive sampling is one of the non-probability sampling techniques. It is proposed to use any kind of probability sampling technique for more generalization of the study. Moreover, the sample of the study was collected from the teachers of the school level. It is proposed to gather sample from other countries of middle east as well. Additionally, the sample consist of Male as gender only. It is suggested that future studies may choose both gender as respondents of the study. Additionally, it is also proposed to develop a proper conceptual framework in future similar studies in the form of independent variables, dependent variable, mediating variable and moderating variable. Conceptual framework will provide more clarity in the research. Furthermore, this study used simple statistical tools for the analysis of data. Future studies may use modern research tools in the future studies in the form of Smart PLS or Process Macros. Additionally, this study used quantitative research methodology in the study. Future studies are encouraged to use mixed methodology or qualitative research methods in their studies.

7. Theoretical and Managerial Implications

The relationship discussed in this study has significant implications for professional and teacher training development. This study will help the teachers to develop metacognitive awareness. The results may provide toolkit that provides the instructors skills to handle students in classroom. Studying empowers teachers so that they become expert in handling curricula rather than implementing curricula. Present study provides an analytical framework and critical conceptual framework to understand pivotal constructs in the education of mathematics, namely metacognitive thinking and mathematical rigor, this research seeks to highlight the symbiotic relationship between thinking and rigor. It is the area that remains unexplored in past studies.

This study is theoretically important as it fills the gap of empirical studies that have investigated variables between mathematics education. Additionally, there is lack of studies that has studied these variables in Middle eastern context. The studies that have discussed these factors from the Asian or Western perspective. So, this study adds to the body of knowledge from this perspective as well.

Research also generates a number of applied benefits for instructional excellence and professional development of teachers. By discussing the connection between metacognition and rigor, the findings can confirm the design of targered training programs. The findings also empower teachers to refine their practices to improve their performance. On the basis of findings, the role of teachers can be refined on the factors of productive richness, strategic diversity, critical contemplation and precision and accuracy. The results contribute to enhance the competence of teacher, enrichment of teaching culture, and development of positive attitude among teachers. Thus, this research serves as significant and novel contribution. This study seeks to fill the gap of limited studies between metacognitive thinking and teachers of mathematics at school level. These results are helpful for the policy makers of education sectors by which they can develop policies for the improvement of their educational sector. Academicians can also use these findings for their future studies.

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