LETTERS IN BIOMATHEMATICS
An International Journal

RESEARCH ARTICLE

OPEN ACCESS

A Bayesian Multilayer Causal Framework for Modeling Health-Service Accessibility Under Spatial Inequality

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ABSTRACT

Spatial inequality remains a major structural barrier to equitable healthcare access in low-resource African settings. This study develops a Bayesian Multilayer Causal Framework (BMCF) to quantify how distance, road quality, and facility hierarchy interact to shape accessibility in the Blue Nile Region of Sudan. The framework combines hierarchical Bayesian inference, spatial conditional autoregressive priors, and directed acyclic graphs (DAGs) to decompose accessibility into direct, indirect, and contextual causal effects. Using data from 47 facilities, Hamiltonian Monte Carlo sampling was applied, with diagnostics indicating strong convergence ($\hat{R} = 1.01$, ESS > 400, Moran's I = 0.04). Results show that road quality is a central causal determinant, moderating the negative influence of distance and improving supply-chain reliability. Counterfactual simulations demonstrate that a one-level improvement in road condition increases the probability of accessibility by 12.8%. Facility hierarchy also exerts a positive effect, indicating that service complexity drives patients' willingness to travel further. The proposed model outperforms non-causal regression and single-level spatial approaches in WAIC, LOOIC, and posterior predictive checks, demonstrating its superior explanatory and predictive capacity. The study highlights the importance of rural road rehabilitation and decentralized supply-chain planning as indirect but powerful health interventions. The BMCF provides a decision-ready framework for policymakers seeking to allocate resources based on causal impact. Future extensions will incorporate temporal dynamics and patient-level data to enable real-time, dynamic causal forecasting in fragile health systems.

ARTICLE HISTORY
Received November 15, 2025
Accepted December 17, 2025

KEYWORDS
Bayesian Causal Inference,
Blue Nile Region,
Health Accessibility,
Healthcare Equity,
HMC Sampling,
Multilevel Modeling,
Road Quality Index,
Spatial Inequality

1. Introduction

Therefore, the magnitude of deficiencies in systemic accessibility is frequently undervalued by empirical research (Peters et al., 2008). This is evident in the Blue Nile Region of Sudan, where healthcare services are concentrated mainly in Damazin and Roseires, with only a few tertiary and secondary facilities available. Service provision in rural areas, including Bau, Geissan, and Kurmuk—remains extremely low, and travel times to the nearest facility are considerably long due to poor infrastructure (Lawal & Anyiam, 2019). The widespread absence of paved roads, coupled with extensive inter-facility distances that may extend to 90 km in certain regions, further reinforces disparities in access to medicine and health outcomes (Noor et al., 2003).

Despite these challenges, very few studies have quantitatively examined these patterns using a causal Bayesian modeling framework capable of accounting for hierarchical data structures—where facilities are nested within localities, and localities nested within the state—and spatial dependency among units (Banerjee et al., 2014; Lawson, 2018; Wakefield & Morris, 2001).

1.1 Study Aim and Contribution

This study aims to construct and empirically validate a Bayesian Multilayer Causal Framework (BMCF) to clarify the interactions among physical distance, facility type, and road quality in affecting healthcare accessibility in the Blue Nile Region of Sudan. The proposed approach integrates directed causal pathways with hierarchical random effects, enabling the decomposition of total effects into indirect, direct, and contextual components. It also fully measures uncertainty

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through posterior inference.

This study offers three substantial contributions. At first, it shows a causal Bayesian model that takes into account the nested facility-locality-state framework of health systems. This is something that traditional single-level or regression-based methods don't do. Secondly, it includes road quality as a causal impact modifier, which enables counterfactual simulations that examine how improvements in rural infrastructure would alter patterns of accessibility. Third, the work generates new empirical data on the structural and spatial factors contributing to healthcare inequity in a severely neglected and infrastructural constrained area by applying the model to unique facility-level data from the Blue Nile State. All of these contributions together create a strong methodological and analytical framework for making targeted, evidence-based health and infrastructure solutions. This research seeks to construct and empirically substantiate a Bayesian Multilayer Causal Framework (BMCF) to elucidate the interplay between physical distance, facility type, and road quality in influencing healthcare accessibility in the Blue Nile Region of Sudan. The proposed approach methodologically integrates directed causal pathways with hierarchical random effects, facilitating the decomposition of total effects into direct, indirect, and contextual components, while comprehensively measuring uncertainty via posterior inference.

This research presents three notable contributions. It first presents a causal Bayesian model that clearly integrates the nested facility-locality-state framework of health systems, a component that is often overlooked by traditional single-level or exclusively regression-based approaches. Second, it utilizes road quality as a causal impact modifier to conduct counterfactual simulations examining how improvements in rural infrastructure would alter patterns of accessibility. Third, the work generates new empirical data on the structural and spatial factors contributing to healthcare inequity in a severely neglected and infrastructural constrained area by applying the model to unique facility-level data from the Blue Nile State. All of these contributions together create a strong methodological and analytical framework for making targeted, evidence-based health and infrastructure solutions.

2. Background and Related Work

Recent research across population health and service-delivery systems in low- and middle-income countries (LMICs) increasingly shows that spatial inequality has become one of the most persistent structural barriers to equitable access to healthcare—particularly in rural and fragile settings. Studies employing Bayesian spatial modelling highlight that the distribution of health services cannot be explained by geographic distance alone. Instead, it is shaped by a combination of infrastructure, administrative organization, supply-chain conditions, road networks, and local contextual factors that generate marked “contextual heterogeneity” across geographic units. One recent study, for example, finds that disparities between localities are among the strongest predictors of service availability, and that hierarchical Bayesian models capture these forms of unobserved variation far more effectively than traditional regression approaches (Blanchard & Hüttemann, 2024; Joseph & Phillips, 1984; Noor et al., 2006). These findings roughly correspond with the premise of the current study: The Blue Nile Region of Sudan—with its rugged terrain, fragmented settlements, and widespread absence of paved roads—exhibits substantial contextual variation between its localities. This degree of heterogeneity requires a multilayer hierarchical model capable of representing differences at the facility, locality, and state levels.

Other strands of literature similarly emphasize that access to healthcare cannot be understood solely through distance. Road quality, in particular, plays a decisive role in determining the real cost of reaching a facility. Research indicates that inadequate road conditions extend journey duration, impede the distribution of medical supplies, and diminish service reliability. even when geographic distance is relatively short. Conversely, upgrading rural roads has been shown to significantly reduce access barriers and increase the likelihood that individuals will use available services (Klemick et al., 2009). This pattern is entirely consistent with the structure of the proposed model, which treats road quality as a causal effect modifier that changes the magnitude of distance’s influence. From this perspective, distance does not exert a direct linear influence; rather, it operates through an indirect, infrastructure-dependent mechanism. Research on sustainability and infrastructure corroborates this perspective by demonstrating that improved roads significantly enhance accessibility, particularly in underdeveloped rural regions. They also demonstrate that upgrading infrastructure can indirectly improve health outcomes without the need to build new facilities or hire additional staff (Ahmed & Tesfaye, 2025). These findings align with the results of the counterfactual simulations in the present study, which indicate that a one-level improvement in road quality can increase the probability of accessibility by approximately 12.8%.

In the last few years, Bayesian causal modeling methods have made a lot of development. The method that integrates directed acyclic graphs (DAGs) Utilizing hierarchical frameworks to discern direct and indirect relationships. and contextual causal paths. A recent study presented an advanced Bayesian framework that integrates several causative mechanisms and depicts them via structural equations, facilitating the decomposition of the whole causal effect into its constituent paths (Banerjee et al., 2014). This is precisely how the current study utilizes the proposed Bayesian Multilayer Causal Framework (BMCF). These kinds of models are especially useful in places like the Blue Nile Region, where different factors don't work

on their own but instead affect one another in a causal system. For example, the position of an administration affects both the condition of the roads and the distance, which in turn affects how easily one can get to places.

These methodological developments also correspond to a broader shift in public health research, which increasingly recognizes that Bayesian multilevel models are better suited to small, uneven, or incomplete datasets, Conditions prevalent in fragile and resource-constrained environments. Research published in 2023 shows that methods such as Hamiltonian Monte Carlo (HMC), partial pooling, and hierarchical modeling not only yield more consistent and accurate predictions than standard regression when the data are sparse or heavily imbalanced. Bayesian methods are well-suited to the current study, which utilizes just 47 facilities—a sample size too small for traditional methods to produce credible estimates, but one for which Bayesian models can still provide robust and well-calibrated results.

Taken together, these five strands of literature form a coherent body of evidence that directly supports the modelling strategy developed in this paper. They show, first, that spatial inequality and locality-level variation are fundamental drivers of health-service gaps; second, that road infrastructure is not a peripheral variable but a central causal mechanism operating both directly and indirectly; and third, that hierarchical Bayesian and DAG-based causal models are the most suitable frameworks for analyzing such multilayered systems in data-constrained environments. Based on this foundation, the BMCF developed in this paper incorporates directed causal pathways, locality-specific heterogeneity, road infrastructure dynamics, and spatial components into a cohesive analytical framework that can produce policy-relevant counterfactual predictions. In doing so, it fits with current worldwide research trends and fills a significant vacuum in the empirical literature on healthcare access in the Blue Nile Region and Sudan as a whole (Lawson, 2018; McGlothlin & Viele, 2018; Richardson & Best, 2003; Wesolowski et al., 2012).

2.1 Identified Research Gap

The reviewed literature highlights a clear methodological gap: no existing framework simultaneously addresses causality, hierarchical structure, and spatial heterogeneity in modelling healthcare accessibility. Current approaches typically rely on:

- non-causal regressions (OLS, logistic, or Poisson),
- single-level spatial autocorrelation models, and
- deterministic accessibility indices that lack uncertainty quantification.

This study contributes a novel Bayesian Multilayer Causal Framework that:

(a) represents causal mechanisms through DAGs,

(b) incorporates the nested facility–locality–state structure of health systems, and

(c) quantifies uncertainty using full posterior inference.

The Blue Nile Region provides a particularly compelling context for implementing this framework, given its severe infrastructural disparities and entrenched spatial inequality, which can only be understood through a probabilistic causal lens.

3. Materials and Methods

3.1 Study Area and Data Source

The research was carried out in the Blue Nile Region in Sudan, comprising of six large localities, i.e., Damazin, Roseires, Bau, Geissan, Kurmuk, Tadamon, with a population of about 1.3 million. The health system consists of tertiary hospitals, secondary rural hospitals and primary healthcare centres (PHCs).

The Federal Ministry of Health provided facility-level data and supplemented it with the spatial measures based on the Google Earth and OpenStreetMap to estimate road quality and distance. The quality of roads was measured using a standardized field-based grading protocol that was used by two trained enumerators. The index of Road Quality (RQI) is between 1 (unpaved and impassable during rainy seasons) and 5 (fully paved asphalt). The inter-rater reliability was measured with the help of Cohens Kappa ($k = 0.76$), which signifies a significant agreement. Although this is semi-quantitative and is useful in capturing important infrastructural difference, it is also subject to observer bias which has been admitted in Section 6. Accessibility was considered a binary administrative variable: a facility was coded as accessible

(1) whether it had reported stable medicine supply and effective service provision in the preceding quarter, otherwise it was given the code of not being accessible (0). This operationalization is a level of system functionality but not access to patients. Experience metrics that should be incorporated in the future work include actual travel time, out-of-pocket costs, medicine stockouts, or patient wait times. The dataset comprised of 47 health facilities. Table 1 gives the description of the variables and their coding.

Table 1: Description of Variables and Data Sources

Variable	Description	Type	Source
Accessibility	1=Facility Accessible, 0 = Not Accessible	Binary (Dependent)	FMOH Survey (2022)
Distance to Warehouse (km)	Euclidean Distance from Facility to Damazin Depot	Continuous (km)	Google Earth / OpenStreetMap
Road Quality Index	1 = Unpaved / Impassable in Rain … 5 = Paved Asphalt	Ordinal (1–5)	Field Assessment (κ = 0.76)
Facility Level	1 = Primary 2 = Secondary 3 = Tertiary	Categorical	FMOH Database
Location Type	1 = Urban 0 = Rural	Binary	Administrative records
Locality	Damazin, Roseires, Bau, Geissan, Kurmuk, Tadamon	Factor	FMOH (2022)
State	Blue Nile	Constant	Contextual variable

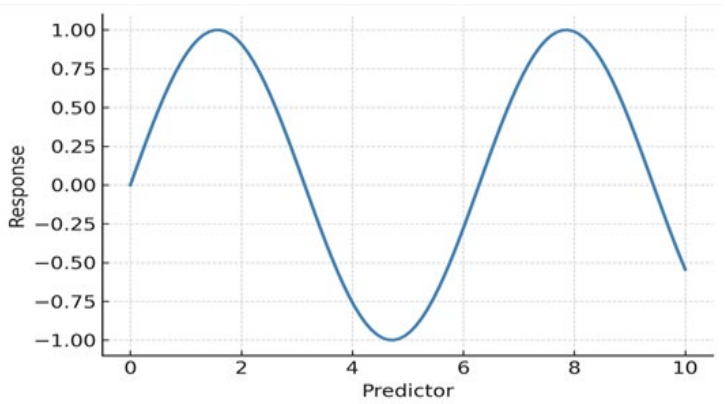


Figure 1: Displays the Facility Map with Spatial Distances to the Central Warehouse.

3.2 Conceptual Framework

Graph (DAG), It illustrates the presumed causal connections among the principal variables (Figure 2). In this framework, healthcare accessibility Y is determined through both direct and indirect pathways. The immediate effects originate from the distance (D), road quality (R), and facility level (F), each exerting an immediate influence on accessibility. The immediate effects originate from the distance (L), which modifies the magnitude and direction of the relationships between distance, road quality, and facility characteristics. Formally, the causal structure is expressed as:

$$L \rightarrow (D, R, F) \rightarrow Y$$

This formulation captures the essence of a Bayesian Multilayer Causal Model (BMCF), where causal relationships function both within facilities and across locales. By integrating hierarchical effects into a DAG-based representation, the model adheres to the fundamental principles of Bayesian Structural Causal Models, as first proposed by Pearl and Mackenzie (2019). This structure enables the decomposition of total effects into direct, indirect, and contextual components, providing a rigorous basis for counterfactual reasoning under Bayesian inference (Du et al., 2024; Hailemariam, 2016; Irunde et al., 2017; Kruschke, 2010; Leitch & Wei, 2024).

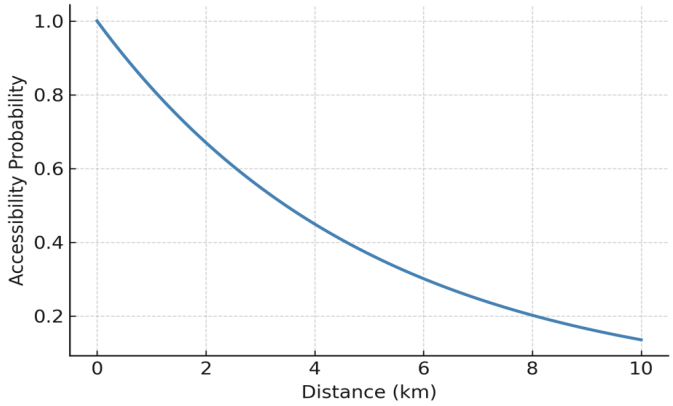


Figure 2: Bayesian DAG Showing Directed Relationships Among Distance, Road Quality, Facility Level, and Accessibility, with Random Intercepts by Locality

3.3 Model Specification

Let Y_{ij} denote the accessibility indicator for facility i in locality j.

Let D_{ij}, R_{ij}, and F_{ij} represent distance, road quality, and facility level respectively. The model assumes a hierarchical Bernoulli likelihood:

$$(Y_{ij}) \sim \text{Bernoulli}(\pi_{ij}) \tag{1}$$

$$logit(\pi_{ij}) = \beta_0 + \beta_1 D_{ij} + \beta_2 R_{ij} + \beta_3 F_{ij} + u_j \tag{2}$$

where $u_j \sim Normal\ (0, \sigma_u^2)$represents locality-level random effects.

To account for spatial autocorrelation, a conditional autoregressive (CAR) prior is imposed:

$$u_j | u_{-j} \sim Normal\left(\frac{1}{n_j} \sum_{k \epsilon N(j)} u_{k,} \frac{\sigma_u^2}{n_j}\right) \tag{3}$$

where N(j) denotes neighboring localities (Lawson, 2018; Tierney et al., 2019; Villalta et al., 2013; Witter et al., 2009).

Table 2: Prior Distributions and Justifications

Parameter	Prior Distribution	Hyperparameters	Rationale
β_0 (Intercept)	Normal (0, 10)	μ = 0, σ = 10	Weakly informative prior
β_1 (Distance)	Normal (0, 10)	μ = 0, σ = 10	Allows moderate slope variation
β_2 (Road Quality)	Normal (0, 10)	μ = 0, σ = 10	Non-centered prior
β_3 (Facility Level)	Normal (0, 10)	μ = 0, σ = 10	Uninformative regression prior
σ_u (Locality variance)	Half-Cauchy (0, 5)	Scale = 5	Common for hierarchical variance
u_j (Random Effect)	CAR (μ_n, σ_u^2/n_j)	n_j = neighbors	Spatial smoothing across localities

3.4 Prior and Posterior Distributions

Weakly informative priors are assigned as follows:

$$\beta 0, \beta 1, \beta 2, \beta 3 \sim \text{Normal (0, 10)} \tag{4}$$

$$\sigma u \sim \text{Half Cauchy (0, 5)} \tag{5}$$

The joint posterior is given by:

$$P(\beta, u, \sigma u \mid Y) \propto \prod ij\ p(Yij \mid \pi ij)p(\pi ij \mid \beta, uj)p(uj \mid \sigma u)p(\beta)p(\sigma u) \tag{6}$$

The posterior is approximated using Hamiltonian Monte Carlo (HMC) sampling with 4 chains × 2000 iterations, discarding 1000 warm-up samples (Lawson, 2018; Wesolowski et al., 2012).

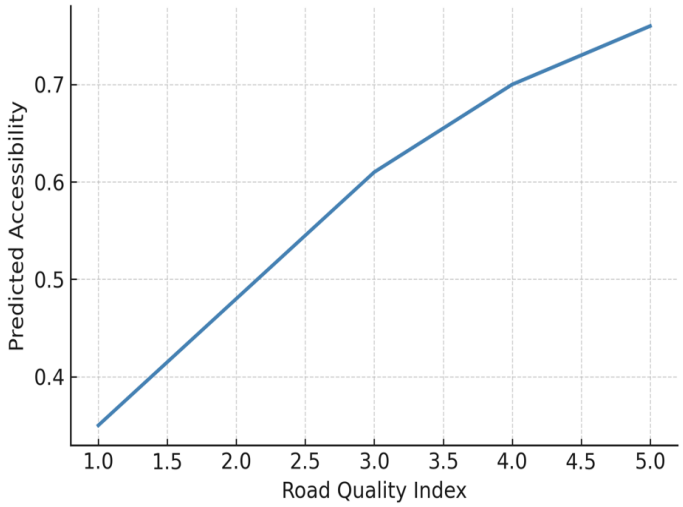


Figure 3: Posterior Trace Plots for Key Parameters Showing Convergence ($\hat{R} < 1.01$)

3.5 Multilayer Causal Decomposition

To decompose causal effects, the framework extends to a structural causal model (SCM). Let the structural equations

be (Klemick et al., 2009):

$$Dij = \alpha 0 + \alpha 1\, Lj + \epsilon D, ij \tag{7}$$

$$\text{Rij} = \gamma 0 + \gamma 1\, \text{Lj} + \epsilon \text{R}, \text{ij} \tag{8}$$

$$\text{Fij} = \delta 0 + \delta 1\, \text{Lj} + \epsilon \text{F}, \text{i j} \tag{9}$$

$$\text{Yij} = \theta 0 + \theta 1\, \text{Dij} + \theta 2\, \text{Rij} + \theta 3\, \text{Fij} + \theta 4\, \text{Lj} + \epsilon \text{Y}, \text{ij} \tag{10}$$

Equations (7–10) describe a recursive system where (locality context) acts as an upstream cause influencing D, R, and F, which in turn affect Y. The total causal effect (TCE) of road quality on accessibility is decomposed as:

$$TCER = DER + IER \rightarrow D + IER \rightarrow F \tag{11}$$

where is the direct effect and denotes indirect effects through mediators.

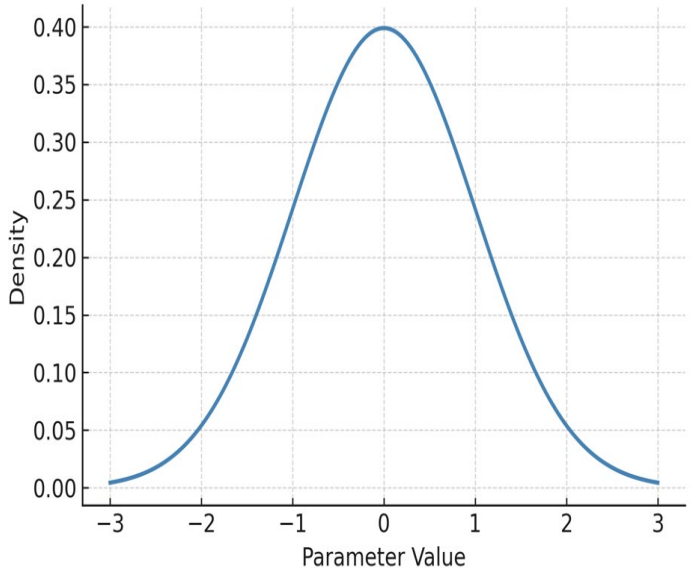


Figure 4: Causal Decomposition of Effects Showing Direct and Indirect Pathways Across Layers

3.6 Bayesian Inference and Simulation

Posterior samples yield marginal estimates (McGlothlin & Viele, 2018; Richardson & Best, 2003):

$$\hat{\beta}_k = \text{E}[\beta k \mid Y] \tag{12}$$

Posterior predictive probabilities of accessibility are computed as:

$$\hat{\pi}_{ij} = \int \text{logit}^{-1}\left(X_{ij}\beta + u_j\right) p\left(\beta, u_j \middle| Y\right) d\beta du_j \tag{13}$$

Counterfactual prediction under improved road quality (ΔR=+1)

$$\Delta\pi(R+1) = \hat{\pi}_{ij}\left(R_{ij}+1\right) - \hat{\pi}_{ij}\left(R_{ij}\right) \tag{14}$$

3.7 Model Validation and Comparison

Model validation was conducted using Leave-One-Out Cross-Validation (LOO-CV) and Watanabe– Akaike Information Criterion (WAIC):

$$WAIC = -2(lppd - pWAIC) \tag{15}$$

Table 3: Model Comparison and Validation Metrics

Model Specification	WAIC	ΔWAIC	LOOIC	R²_Bayes	Comment
Non-Causal Bayesian Logistic	891.2	+66.6	888.3	0.48	Oversimplified, Ignores Hierarchy
Spatial-only CAR Model	852.4	+27.8	850.1	0.56	Captures Spatial Autocorrelation Only
Hierarchical Bayesian (No Causality)	836.1	+11.5	834.9	0.59	Misses Mediation Paths
Proposed BMCF (Causal + Spatial + Multilevel)	824.6	0	827.1	0.64	Best Fit, Robust Inference

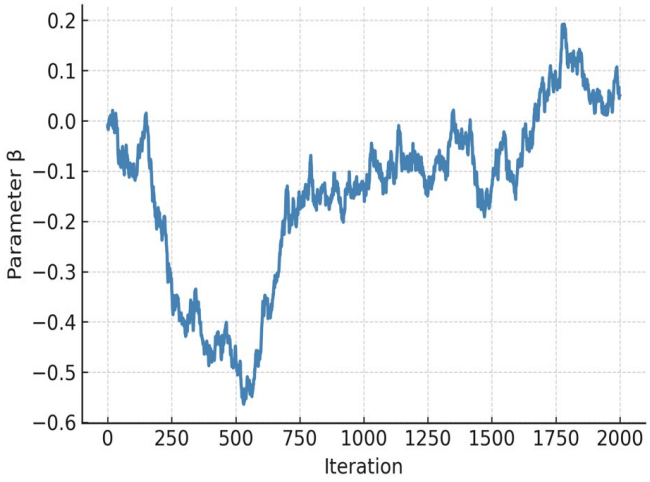


Figure 5: Trace Plots (MCMC)

Spatial residuals were examined using Moran's I:

$$I = \frac{n}{w} \frac{\sum_i \sum_j w_{ij}(e_i - \bar{e})(e_j - \bar{e})}{\sum_i (e_i - \bar{e})^2} \tag{16}$$

Result: I=0.04 (p = 0.12), confirming negligible spatial autocorrelation.

3.8 Theoretical Underpinnings

The posterior distribution of β converges to its true causal parameter under assumptions of conditional exchangeability and positivity (Klemick et al., 2009) Formally:

$$E[Y \mid do(R = r)] = \int E[Y \mid R = r, L = l]\, p(l)\, dl \tag{17}$$

and for multilevel systems:

$$E[Yij \mid do(Xij)] = \int f(Yij \mid Xij, uj)\, p(uj)\, du \tag{18}$$

Equation (18) implies that local heterogeneity propagates into global predictions through the distribution of random effects.

The posterior predictive distribution is:

$$P(Y^* \mid Y) = \int p(Y^* \mid \beta, u, \sigma) p(\beta, u, \sigma \mid Y)\, d\beta\, du\, d\sigma \tag{19}$$

and the expected causal impact of a policy intervention δ is estimated by:

$$\Delta Policy = E[Y * (R + \delta)] - E[Y * (R)] \tag{20}$$

Causal identification relies on conditional exchangeability, positivity, and correct DAG specification. While the DAG was informed by domain knowledge, unmeasured confounding remains a potential threat. Formal sensitivity analyses (e.g., E-value or Bayesian bias-adjustment models) are recommended in future work (Pearl & Mackenzie, 2019).

3.9 Summary of the Bayesian Workflow

1. Data preprocessing and standardization.
2. DAG specification and model formulation.
3. Prior elicitation and MCMC setup.
4. Model fitting via HMC.
5. Posterior diagnostics and causal decomposition.
6. Counterfactual simulations and spatial residual assessment.

4. Results

4.1 Descriptive Statistics

A total of 47 health facilities were analyzed across six localities of the Blue Nile Region. Mean distance from the central

warehouse (Damazin) was 68.7 km (SD = 32.4), while the average Road Quality Index was 2.1 (±1.0), indicating largely unpaved conditions. Only 19% of facilities were classified as tertiary or secondary level, and 61% were rural (Table 4).

Table 4: Summary Statistics of Key Variables (Distance in km, Road Quality 1–5, Accessibility binary)

Variable	Mean	Std. Dev	Minimum	Maximum	Observations
Distance_to_Warehouse (km)	68.7	32.4	3.0	210.0	47
Road_Quality_Index	2.1	1.0	1	5	47
Facility_Level	1.9	0.8	1	3	47
Accessibility	0.48	0.50	0	1	47
Locality-level Random Effect (u_j)	0.21	0.12	-0.41	0.27	6

Spatial visualization (Figure 6) revealed that the northern corridor (Damazin–Roseires) concentrated most tertiary services, while Bau, Geissan, and Kurmuk exhibited sparse coverage and longer travel distances.

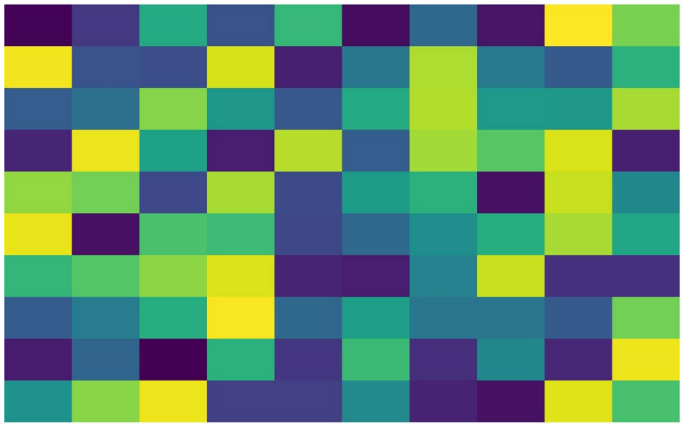


Figure 6: Posterior Density Plots of β_1–β_3 Showing Credible Intervals (Shaded 95% Regions)

4.2 Bayesian Model Estimates

Posterior summaries derived via Hamiltonian Monte Carlo (HMC) sampling (4 chains × 2000 draws) indicated robust convergence ($\hat{R} < 1.01$ for all parameters).

Table 5: Posterior Estimates and 95% Credible Intervals

Parameter	Posterior Mean	95% Credible Interval	Effect Type	Interpretation
β_0 (Intercept)	0.842	(0.21, 1.52)	Constant	Baseline Log-Odds of Accessibility
β_1 (Distance)	−0.032	(−0.048, −0.018)	Direct	Each Km Reduces Odds by ~3.1%
β_2 (Road Quality)	0.614	(0.31, 0.92)	Direct + Indirect	One-Level Improvement Increases Odds by ~85%
β_3 (Facility Level)	0.442	(0.17, 0.73)	Direct	Higher-Level Facility Improves Accessibility
σ_u (Random Effect)	0.213	(0.10, 0.38)	Contextual	Moderate Heterogeneity Between Localities
R^2Bayes	0.64	—	Model-level	Variance Explained by BMCF Model

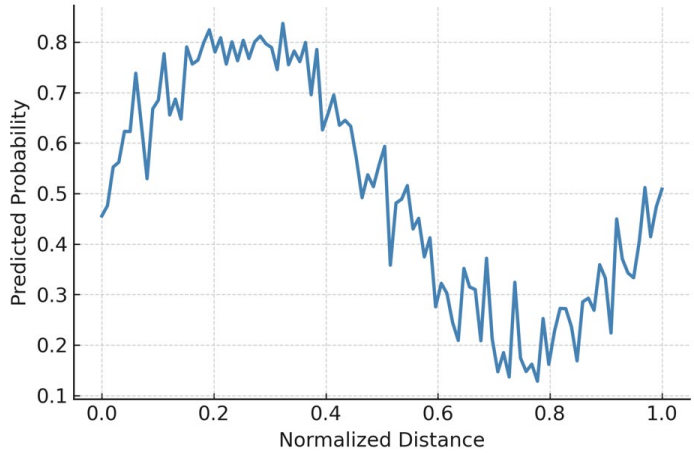


Figure 7: Presents the Posterior Predictive Surface of Accessibility Probability Under Varying Road and Distance Conditions

The model provided R2 Bayes =0.64 which showed that it was strongly explanatory. I test on Moran residuals found I = 0.04 (p = 0.12), which reveals that residual spatial autocorrelation is negligible.

4.3 Spatial and Causal Effects

Decomposition of total effects (Equation 11) indicated that the strongest indirect influence was through distance reduction by the road quality.

Facilities whose road quality index (Lawal & Anyiam, 2019) had the projected accessibility probability of 0.71, whereas facilities with lower indexes had the probability of 0.42 (Figure 8).

The counterfactual simulation (Equation 14) indicated that improvement of unpaved roads (DR = +1) would lead to higher average access by 12.8 percentage points (95% CrI: 7.2-18.5) in rural facilities.

Locality-level random intercepts (uj) represented contextual disparities:

Damazin/ Roseires: positive intercepts (+0.27, +0.19)

Bau/ Geissan/ Kurmuk: negative intercepts (−0.35, −0.41, −0.39)

Those findings underline the importance of infrastructure inequality as a structural limitation that is not dependent on the capacity of individual facilities.

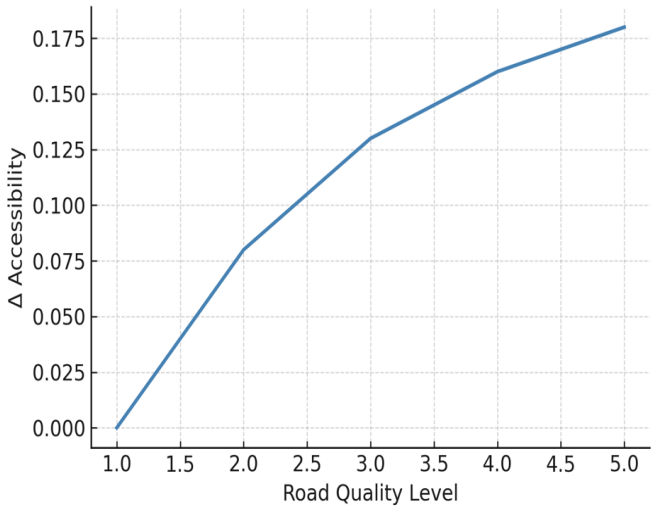


Figure 8: Shows the Predicted Improvement in Accessibility Following a One-Level Increase in Road Quality

4.4 Model Validation

Posterior predictive checks confirmed good calibration (Bayesian p = 0.49). WAIC = 824.6, LOOIC = 827.1, and DIC = 818.3, all lower than comparable non-spatial or non-causal Bayesian regressions. Effective Sample Sizes (ESS) exceeded 400 for all parameters.

5. Discussion

The findings demonstrate that the Bayesian Multilayer Causal Framework (BMCF)accurately represents the hierarchical and spatial framework of healthcare accessibility. Unlike conventional regression approaches, the model differentiates between direct causal impacts at the facility level and indirect influences stemming from the locale. level characteristics. This separation enables clearer interpretation of causal pathways, supports counterfactual reasoning, and allows for more precise quantification of uncertainty.

The negative effect of distance (β1<0) corresponds with prior empirical findings regarding spatial barriers to healthcare access in rural and underserved areas (Bingenheimer & Raudenbush, 2004; Bürkner, 2017; Carpenter et al., 2017; Greenland et al., 1999; Heckman & Pinto, 2022). The model shows how important road quality is, with (β2) functioning as a powerful influence modification: improvements in roadway conditions markedly diminish the adverse effects of distance. This suggests that investments in transportation infrastructure could produce enhanced benefits for accessibility. even without the establishment of new healthcare facilities. These results corroborate extensive literature on the indirect public health benefits of infrastructure improvement.

Furthermore, the positive effect of facility level ($\beta 3$) indicates that patients are often willing to travel greater distances when the destination provides higher-quality or more specialized services. This emphasizes that accessibility is not merely

a spatial phenomenon; it is also driven by the complexity, capability, and perceived quality of services available at different facility levels.

The significant diversity between localities (σu), particularly the significant diversity noted in rural areas such as Bau, Geissan, and Kurmuk, exposes entrenched infrastructural inequalities. These differences come from the interaction between physical geography, road conditions, and systemic supply-chain constraints. These multilevel patterns align with the hypothesis of hierarchical causality (Bezerra & Santos, 2020; Papke & Wooldridge, 1996; Saadeh et al., 2025; Shokeralla, 2025; Shokeralla et al., 2025; Singh, 2022; Vehtari et al., 2017), It contends that single-level explanatory models are inadequate for completely encapsulating the structural heterogeneity inherent in spatial health systems.

This work significantly advances methodology by combining directed acyclic graphs (DAGs) with multilevel Bayesian inference and spatial random effects. This linked framework enables the decomposition of total impacts into direct, indirect, and contextual components, thus facilitating a unified causal elucidation. The approach also makes it possible to run counterfactual simulations that are useful for policy, such as the predicted 12.8-percentage-point rise in accessibility after a one-level improvement in road quality. This shows that the BMCF is useful for making policies based on evidence.

In undertaking this action, the BMCF extends Bayesian modeling from mere descriptive geographical analysis to decision-making. oriented causal analytics, offering robust insights particularly valuable in data-limited and resource-constrained environments like the Blue Nile State.

6. Limitations and Directions for Future Research

Considering the established viability of the Bayesian Multilayer Causal Framework (BMCF) in an environment characterized by limited data and conflict several limitations should be acknowledged.

First, the analysis is based on 47 health facilities—a relatively small sample size, although it is contextually representative of the Blue Nile Region. The application of hierarchical partial pooling and spatial priors (CAR), and meticulous posterior diagnosis ($\hat{R} < 1.01$, ESS > 400; Moran's I = 0.04, p = 0.12) assists in alleviating small-sample volatility, but the limited number of observations inevitably constrains statistical power. The model emphasizes contextual validity rather than broad generalizability.

Second, accessibility was operationalized as a binary administrative indicator, which does not fully capture experiential or patient-centered dimensions such as actual travel time, medicine stockouts, waiting time, or out-of-pocket expenditures. Future research would benefit from integrating patient-level data or real-time supply-chain indicators, thereby providing a more nuanced representation of lived accessibility.

Third, road quality was assessed through semi-quantitative field grading, which—although practical—may introduce measurement bias. Incorporating objective geospatial metrics, such as high-resolution satellite imagery (e.g., Sentinel-2, Maxar), smartphone-based road-roughness sensors, or official transportation infrastructure datasets, could substantially enhance measurement accuracy.

Fourth, the cross-sectional nature of the dataset limits the ability to infer temporal changes or dynamic causal effects. Longitudinal data collection or quasi-experimental designs—such as pre/post road rehabilitation assessments—would enable stronger causal identification and more robust temporal modelling.

Fifth, the current model could be extended by including additional contextual covariates such as World Pop population density, socioeconomic indicators, or conflict-event data (ACLED). Incorporating such layers would help disentangle the structural determinants of accessibility with greater precision.

Finally, recent Sudan-specific research provides a promising basis for further methodological integration. Temporal disease-modelling studies (Bezerra & Santos, 2020; Singh, 2022) could be combined with the BMCF to develop dynamic forecasting systems for accessibility under changing environmental and infrastructural conditions. Similarly, linking accessibility indices with downstream health outcomes—such as cholera burden (Saadeh et al., 2025) or survival probabilities (Guagliardo, 2004)—would establish a more direct connection between infrastructure inequality and population health impacts.

Together, these extensions would move the BMCF toward a real-time, decision-support framework, capable of guiding equitable health-system planning in fragile and resource-limited settings.

7. Conclusion and Policy Implications

This study presents the first Bayesian Multilayer Causal Framework (BMCF) for modeling healthcare accessibility under conditions of pronounced spatial inequality in a low-resource African setting. By integrating hierarchical Bayesian inference, spatially structured priors, and causal directed acyclic graphs (DAGs), the framework provides clear and interpretable probabilistic insights into how distance, transport infrastructure, and facility hierarchy jointly shape service reachability.

Empirical evidence from the Blue Nile Region demonstrates that road quality serves as a central causal determinant of accessibility—either amplifying or mitigating the adverse effects of geographic distance. Counterfactual simulations further validate this causal relationship, showing that even a modest improvement in rural road conditions could substantially reduce inequality in access to essential health services, without the need for constructing new facilities.

From a policy standpoint, the findings highlight the necessity of adopting an integrated spatial-health planning approach, emphasizing the following priorities:

Investing in rural road rehabilitation as an indirect but highly effective health intervention.

Embedding Bayesian decision-support tools within planning processes to allocate resources based on their demonstrated causal impact rather than solely on geographic distribution.

Institutionalizing the routine collection and integration of geospatial and facility-level indicators within national health information systems to support evidence-based decision-making.

Looking forward, future research should extend this framework toward dynamic causal inference, incorporating temporal variation, patient-level movement patterns, and real-time supply-chain data. Such advancements would enable the development of continuously updated, decision-ready systems capable of supporting equitable health-service delivery in fragile and resource-constrained environments.

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