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# Canine distemper outbreak modeled in an animal shelter 

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#### Abstract

Canine distemper virus (CDV) is a highly contagious virus that can cause outbreaks, specifically in crowding situations, such as an animal shelter, in which a large number of susceptible dogs are brought together. Introduction of this virus into a shelter can have devastating effects, potentially resulting in shelter canine depopulation. Motivated by recent outbreaks in Tennessee, a mathematical model was constructed to find relevant factors that could assist in preventing or reducing outbreaks. A system of ordinary differential equations was derived to represent the spread of CDV through susceptible, exposed, infected and recovered ( $S-E-I-R$ ) classes as well as a vaccinated $(V)$ class. Our model was adapted to represent a local Knoxville shelter. The effects of various control methods, both preventative and corrective, on disease spread were investigated.


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## 1. Introduction

Canine distemper virus (CDV) is a member of the Morbillivirus subfamily of the Paramyxoviridae virus family and is said to be closely related to measles in humans (Greene \& Appel, 2006). CDV is highly contagious and affects a wide range of animals including, but not limited to, the families Canidae, Procyonidae and Mustelidae (Kapil \& Yeary, 2011; Larson \& Schultz, 2006). The first vaccine for CDV was developed in the 1950s and is still used today (Martella, Elia, \& Buonavoglia, 2008).

An infected animal's bodily fluids and aerosols act as a vector for the disease (Litster, Nichols, \& Volpe, 2012). Animals that are infected with CDV initially display respiratory and gastrointestinal symptoms which are then followed by neurological symptoms. The respiratory and gastrointestinal symptoms include heavy breathing, nasal discharge, ocular discharge, vomiting and diarrhoea. The respiratory signs are often misdiagnosed as kennel cough. The neurological symptoms are seizures, blindness, paralysis and involuntary movements. Other symptoms include thickening of the skin on the footpads and nose (Matthews, 2011).

[^0]Following a latent period of 7 days in which a dog displays no symptoms and is not infectious, a dog develops clinical symptoms and is highly contagious for a period of 2 weeks (Greene \& Appel, 2006). Following the acute manifestation of clinical signs, a dog can either recover, develop acute encephalitis and die, or develop chronic encephalitis and eventually die (Krakowka \& Koestner, 1976). The dogs that develop chronic encephalitis experience a symptom-free period that can last up to a few months but during which they are mildly contagious; these dogs gradually come to exhibit neurological symptoms, leading to death (Krakowka \& Koestner, 1976). It has been found that the susceptibility of dogs to CDV varies based on the age of the dog. Younger dogs are more susceptible than older dogs (Krakowka \& Koestner, 1976).

The American Animal Hospital Association's 2011 guidelines state that puppies (under 16 weeks of age) should be boostered every 3-4 weeks until they reach 16 weeks of age, and that adult dogs ( 16 weeks and older) should only receive one vaccine for CDV (Welborn et al., 2011). These recommendations are in place because the modified live vaccine is potent enough to protect adults without maternal antibodies and to protect puppies once they reach the adult age of 16 weeks (hence, the boosters up until this age). Through our research, we have found that shelters follow slightly different vaccination protocols. In a shelter situation, adult dogs are usually boostered once for two reasons. The first being that it is hard to correctly judge the age of a dog as it comes into the shelter, so the booster allows room for error. Secondly, adult dogs are boostered in a shelter because the shelter environment puts the dogs at a higher risk of infection. Another difference within a shelter is the time period: shelters booster after 2 weeks instead of the recommended 3-4 weeks. This is again because the shelter is a higher risk environment for infection, so they want to booster the dogs as quickly after the initial vaccine as possible. However, they cannot booster any earlier than 2 weeks because the dogs' immune systems need a proper amount of time to respond to the initial vaccine before a booster will be effective.

Researchers have modelled the spread of CDV previously for varying reasons, including to investigate the relative impact of vaccination policies on wildlife species affected by CDV outbreaks, including lions and wild dogs (Prager, Woodroffe, Cameron, \& Haydon, 2011; Roelke-Parker et al., 1996). Other models have looked into wildlife species interacting with reservoirs of the virus, such as domestic dog populations, using both contact network and agent-based modelling (Belsare \& Gompper, 2014; Craft, Volz, Packer, \& Meyers, 2009). To the best of our knowledge, no models exist that look at the spread of an outbreak in a shelter situation.

Motivated by recent outbreaks in Tennessee and other Southern states (Riley \& Wilkes, 2015), a mathematical model was formulated to observe the spread of a CDV outbreak in a shelter and find relevant factors that could assist in preventing or reducing outbreaks. This epidemiological S-E-I-R model is a system of ordinary differential equations (ODEs) and provides insights into preventive measures that a shelter can take in order to diminish the negative impacts of a CDV outbreak.

Veterinarians at two local shelters provided two data-sets as well as insights into parameter values, allowing us to model an outbreak in a shelter environment. We then modelled various preventative measures in order to determine what aspects of a shelter's policies are instrumental in the outcome of an outbreak.

In the next section, the aspects of the epidemiological models are explained in detail. Section 3 presents the results from the local shelter application. In Section 4, additional


Figure 1. This diagram shows the possible flow of a CDV outbreak in a shelter with six classes: $S$ (susceptibles), $E$ (exposed), $V$ (vaccinated), $I_{1}$ (acutely infected), $I_{2}$ (chronically infected) and $R$ (recovered).
results, elasticity of the model's parameters and limitations of the model are discussed. The last section examines the conclusions that can be made from our work and also talks about our future work.

## 2. Epidemiological models

Our model reflects the important progression characteristics of CDV, and thus we have the following six classes. The susceptible class ( $S$ ) contains dogs that have not completed the vaccination schedule (received only a single vaccination on intake into the shelter), while the vaccinated class $(V)$ contains vaccinated dogs that have received a booster vaccine and are considered partially protected. The exposed class $(E)$ reflects the dogs currently in the latent stage of CDV infection (infected but not yet showing clinical signs or shedding the virus and are therefore not able to transmit the disease).

The primary infected class $\left(I_{1}\right)$ reflects the acute onset of the disease: this class encompasses the phase of CDV in which the dogs are displaying clinical symptoms and are highly contagious. From this class, dogs can either die due to acute encephalitis, recover, or progress to the secondary infected class ( $I_{2}$ ), which reflects a chronic, slower progression of CDV. The animals who have survived the CDV infection are in the recovered class $(R)$. The model diagram can be seen in Figure 1, and the description of the parameters is in Tables 1 and 2.

In our model, adoptions occur from all classes in which dogs appear healthy. Death occurs out of the infected classes, representing death due to euthanasia. We assumed that the time frame in which a shelter outbreak occurs progresses rapidly enough to ignore natural death. Transmission of the disease can result when a susceptible or vaccinated dog comes in contact with an acutely or chronically infected dog. Once a dog becomes infected with CDV, the dog enters the exposed class until it begins shedding the virus and then transitions into the infected class. Based on the progression of the disease in the individual, the infected dog either recovers, dies, or becomes chronically infected.

Our system of ODEs to model the spread of CDV through a shelter is as follows:

$$
\begin{align*}
S^{\prime} & =b-S\left[\beta_{1} I_{1}+\beta_{2} I_{2}\right]-\delta S-a_{S} S  \tag{1}\\
E^{\prime} & =S\left[\beta_{1} I_{1}+\beta_{2} I_{2}\right]+V\left[\beta_{3} I_{1}+\beta_{4} I_{2}\right]-a_{E} E-\alpha_{1} E  \tag{2}\\
I_{1}^{\prime} & =\alpha_{1} E-\alpha_{2} I_{1}-d_{1} I_{1}-\gamma_{1} I_{1}  \tag{3}\\
I_{2}^{\prime} & =\alpha_{2} I_{1}-d_{2} I_{2}-\gamma_{2} I_{2}  \tag{4}\\
R^{\prime} & =\gamma_{1} I_{1}+\gamma_{2} I_{2}-a_{R} R  \tag{5}\\
V^{\prime} & =-V\left[\beta_{3} I_{1}+\beta_{4} I_{2}\right]+\delta S-a_{V} V . \tag{6}
\end{align*}
$$

This model assumes that all dogs who develop CDV display clinical symptoms (no sub-clinical disease manifestations) and are able to transmit the disease. Therefore, our model does not consider new infections resulting from asymptomatic infections. Although some data have shown a CDV infection can be asymptomatic (Greene \& Appel, 2006), an interview with a veterinarian revealed that under stressful situations, such as being in a shelter, most dogs would progress to showing clinical signs.

We also assume all dogs who develop CDV are diagnosed correctly. Furthermore, age classes are ignored and therefore all dogs are considered to be equally susceptible. This is a simplification we hope to remove from future models. Lastly, it assumes no contact between dogs in the shelter and wildlife populations, which are potential CDV reservoirs.

To investigate stability of this model during an outbreak, we used the next generation matrix method to derive the basic reproductive number, $R_{0}$, which represents the number of secondary infections resulting from one infected individual. When $R_{0}$ is greater than one, the disease persists, while when it is less than one the disease-free equilibrium is locally stable. In this model, the infected compartments are $E, I_{1}$ and $I_{2}$. Following the explanation given in van den Driessche \& Watmough (2002), we obtain $\mathbf{x}=\left(E, I_{1}, I_{2}, R, S, V\right)$ with $x^{\prime}=\mathscr{F}-\mathscr{V}$ :

$$
\mathscr{F}=\left(\begin{array}{c}
S\left[\beta_{1} I_{1}+\beta_{2} I_{2}\right]+V\left[\beta_{3} I_{1}+\beta_{4} I_{2}\right]  \tag{7}\\
0 \\
0 \\
0 \\
0 \\
0
\end{array}\right) \text { and } \mathscr{V}=\left(\begin{array}{c}
\left(a_{E}+\alpha_{1}\right) E \\
\left(\alpha_{2}+d_{1}+\gamma_{1}\right) I_{1}-\alpha_{1} E \\
\left(d_{2}+\gamma_{2}\right) I_{2}-\alpha_{2} I_{1} \\
a_{R} R-\gamma_{1} I_{1}-\gamma_{2} I_{2} \\
S\left[\beta_{1} I_{1}+\beta_{2} I_{2}+\delta+a_{S}\right]-b \\
V\left[\beta_{3} I_{1}+\beta_{4} I_{2}+a_{V}\right]-\delta S
\end{array}\right) .
$$

By evaluating the Jacobian matrices at the disease-free equilibrium, $S^{*}=\frac{b}{\delta+a_{S}}$ and $V^{*}=\frac{b \delta}{a_{V}\left(\delta+a_{S}\right)}$ with $I_{1}^{*}=I_{2}^{*}=R^{*}=0$, we obtain

$$
F=\left(\begin{array}{ccc}
0 & S \beta_{1}+V \beta_{3} S \beta_{2}+V \beta_{4}  \tag{8}\\
0 & 0 & 0 \\
0 & 0 & 0
\end{array}\right) \text { and } V=\left(\begin{array}{ccc}
a_{E}+\alpha_{1} & 0 & 0 \\
-\alpha_{1} & \alpha_{2}+d_{1}+\gamma_{1} & 0 \\
0 & -\alpha_{2} & d_{2}+\gamma_{2}
\end{array}\right) .
$$

We obtain $R_{0}$ from the spectral radius of $F V^{-1}$, which is given by

$$
\begin{align*}
R_{0} & =\rho\left(F V^{-1}\right) \\
& =\frac{\left[S \beta_{1}+V \beta_{3}\right] \alpha_{1}\left(d_{2}+\gamma_{2}\right)+\left[S \beta_{2}+V \beta_{4}\right] \alpha_{1} \alpha_{2}}{\left(a_{E}+\alpha_{1}\right)\left(\alpha_{2}+d_{1}+\gamma_{1}\right)\left(d_{2}+\gamma_{2}\right)} \\
& =\frac{\left[\frac{b}{\delta+a_{S}} \beta_{1}+\frac{b \delta}{a_{V}\left(\delta+a_{S}\right)} \beta_{3}\right] \alpha_{1}\left(d_{2}+\gamma_{2}\right)+\left[\frac{b}{\delta+a_{S}} \beta_{2}+\frac{b \delta}{a_{V}\left(\delta+a_{S}\right)} \beta_{4}\right] \alpha_{1} \alpha_{2}}{\left(a_{E}+\alpha_{1}\right)\left(\alpha_{2}+d_{1}+\gamma_{1}\right)\left(d_{2}+\gamma_{2}\right)} . \tag{9}
\end{align*}
$$

Each of the four terms in the $R_{0}$ expression represents a different transmission route of the disease: a susceptible or vaccinated dog being infected through an interaction with an acutely or chronically infected dog.

### 2.1. Shelter application model

From a visit to a local shelter in Knoxville, we observed the types of interactions between animals and the quality of the environment in which the animals were kept, and considered possible intervention strategies in the case of an outbreak. We were able to see the adoption floor, holding areas, the intake area and the clinic. For dogs specifically, there is one section on the adoption floor. A few dogs are in rooms where they can be paired with one or more dogs. The rest are housed alone in slightly larger rooms where the pens are separated with cinder block walls. These walls only go halfway up to the ceiling with a metal fence completing the barrier. This works well for smaller dogs; however, bigger dogs are able to jump up and interact with their neighbouring dogs. These dog rooms are also part of 'play groups' where volunteers come in and get up to 25 dogs out at a time to play in an outdoor area. Within the dog area, there is a puppy room specific for dogs under 4 months of age. In this room, they are in walled areas based on their litter. All rooms share the same circulated air.

Within the back of the shelter, dogs are held for many reasons. There is an intake area where animal control is allowed to drop off dogs at any time. Upon intake, all dogs are vaccinated for CDV. Puppies are further boostered every two weeks until they are 16-20 weeks old. Usually puppies leave before the two-week period, meaning they do not receive a booster. Adult dogs get vaccinated on intake, and then receive a booster after two weeks. Dogs are held for 72 h after intake, as part of the legal hold period. This is the time in which owners are legally allowed to pick up their animals. After 72 h , the dogs are legally owned by the shelter. At this point, a medical and behaviour assessment is done to determine if the dogs are adoptable. Once the assessments are completed, adoptable dogs are moved to the adoption floor.

Dogs displaying signs consistent with CDV are tested for the virus. Positive dogs within the shelter are euthanized immediately to avoid a shelter-wide outbreak. The area where the positive dogs are held is isolated, preventing further spread of the disease. The veterinarian then tests some of the susceptible dogs that had interacted with the sick dog based on her assessment of the risk factor for infection.

After touring the shelter, we adapted our model as shown in Figure 2. The ODEs that describe the spread of CDV through this local shelter are:

$$
\begin{aligned}
S^{\prime} & =b-\beta_{S} S I-\delta S-a_{S} S \\
E^{\prime} & =\beta_{S} S I+\beta_{V} V I-a_{E} E-\alpha E
\end{aligned}
$$



Figure 2. This applied shelter diagram shows the flow of a CDV outbreak in a local shelter with four classes: $S$, susceptible, $E$, exposed, $V$, vaccinated and $I$, acutely infected.

$$
\begin{align*}
I^{\prime} & =\alpha E-d I \\
V^{\prime} & =\delta S-\beta_{V} V I-a_{V} V \tag{10}
\end{align*}
$$

At this local shelter, due to the prompt euthanasia of any infected dogs, no dogs recover from being acutely infected or transition into becoming chronically infected. Because of the shelter's policies, no chronically infected or recovered class is needed. Then, using the next generation matrix method, we calculated the basic reproductive number of the shelter model to be:

$$
\begin{equation*}
R_{0}=\frac{\alpha\left(\frac{b}{\delta+a_{S}} \beta_{S}+\frac{\delta b}{a_{V}\left(\delta+a_{S}\right)} \beta_{V}\right)}{d\left(a_{E}+\alpha\right)} \tag{11}
\end{equation*}
$$

## Local shelter model assumptions

One of our main assumptions is that all dogs that become infected with CDV are correctly diagnosed with the disease once they show clinical symptoms, around 14 days postexposure, or within a week of being in the infected class (Greene \& Appel, 2006). At this shelter, all dogs are given a vaccine at intake, and then given a booster vaccine 2 weeks later. We assumed that the vaccinated dogs without the booster have a higher chance of contracting CDV than the boostered dogs. The non-boostered dogs are considered susceptibles in this model, while the dogs who have received both the initial vaccine and the booster are considered to be vaccinated.

## Shelter parameters

Through an interview with veterinarians, two data-sets from local shelters and literature on CDV, we estimated the value of all parameters considering the following facts:

- In the month of May, the shelter received 485 dogs.
- On average, the shelter houses 230 dogs.
- In the month of May, 339 dogs were adopted.
- Once the veterinarian notices the clinical symptoms of CDV, the dog is tested and subsequently euthanized if the test results are positive.


Figure 3. The disease-free diagram used to calculate the adoption rates for the local shelter.

CDV test results are obtained on the day of submission due to the shelter currently having access to a PCR machine on site. From the interview and published data on CDV in domestic dogs, we could calculate the relevant parameters as follows:

## Daily drop-off rate

Since the shelter receives approximately 485 dogs in a month, we know that the daily drop-off rate, $b$, is $\frac{485}{31}$.

## Vaccination rates

At this local shelter, the susceptible dogs are given a booster vaccine and therefore become fully vaccinated after 2 weeks. According to a shelter veterinarian, approximately five dogs are vaccinated per day. Given the initial number of susceptible dogs used in our model, we obtained a vaccination rate, $\delta$, of $\frac{1}{28}$, which results in approximately five dogs being vaccinated each day.

## Adoption rates

The adoption rates of the susceptible and vaccinated dogs, $a_{S}$ and $a_{V}$, are calculated by evaluating the shelter dynamics in a disease-free state and calculating the rates such that in the month, 339 dogs were adopted. In a disease-free state, we get the following simplified system of ODEs describing the dynamics (see Figure 3):

$$
\begin{align*}
S^{\prime} & =b-\delta S-a_{S} S  \tag{12}\\
V^{\prime} & =\delta S-a_{V} V . \tag{13}
\end{align*}
$$

We know that the number of dogs on day 0 is 230 dogs. Given the shelter information, the initial populations of susceptible and vaccinated dogs were estimated to be $S_{0}=147$ and $V_{0}=83$, respectively. We also know that the total number of dogs adopted in May was 339 . Therefore, the final population after the month of May is equal to the initial population plus the dropped-off dogs (but without the adopted dogs) or $S(31)+V(31)=$ $S_{0}+V_{0}+485-339$, which leads to $S(31)+V(31)=230+146=376$. From this, we use the system of ODEs to solve for values of $a_{S}$ and $a_{V}$, given the initial population values. We obtain the following explicit equations for $S(t)$ and $V(t)$ using (17) and (18), and assuming that $S(0)=S_{0}$ and $V(0)=V_{0}$ :

$$
\begin{align*}
S(t)= & \frac{b}{a_{S}+\delta}+\left(S_{0}-\frac{b}{a_{S}+\delta}\right) e^{\left(-a_{S}-\delta\right) t} \\
V(t)= & \frac{b \delta}{a_{V}\left(a_{S}+\delta\right)}+\frac{\delta\left(S_{0}-\frac{b}{a_{S}+\delta}\right) e^{\left(-a_{S}-\delta\right) t}}{a_{V}-a_{S}-\delta} \\
& +\left[V_{0}-\frac{b \delta}{a_{V}\left(a_{S}+\delta\right)}-\frac{\delta\left(S_{0}-\frac{b}{a_{S}+\delta}\right)}{a_{V}-a_{S}-\delta}\right] e^{-a_{V} t} \tag{14}
\end{align*}
$$

Using the above expressions for $S(t)$ and $V(t)$ as well as the value for $S(31)+V(31)$ and the initial distribution of susceptible and vaccinated dogs, given an assumption about the relationship between $a_{V}$ and $a_{S}$, we use MATLAB to solve for the adoption rate parameters. We know that at this shelter, the adoption rate for susceptible dogs is much higher than that of vaccinated dogs. In order to get the correct average number of dogs boostered daily at this shelter, we determined the relationship that $a_{S}=3 * a_{V}$ which leads to values of $a_{S}=.0504$ and $a_{V}=.0168$. We assumed that the exposed adoption rate is the average between the other two adoption rates and obtained $a_{E}=.0336$.

## Disease transmission rates

With data from a local shelter, we estimated the disease transmission rate $\beta$ for susceptible dogs. The local shelter we are modelling provided data indicating that they vaccinate their dogs for CDV on intake. This data included the number of infected dogs as well as the time frame of the disease spread. To find the transmission rate, we started by calculating the intrinsic growth rate using an approximate equation describing the disease dynamics at the beginning of an outbreak,

$$
\begin{equation*}
I(t)=I(0) e^{\lambda t} \tag{15}
\end{equation*}
$$

Given this relationship with the data, we found that

$$
\begin{equation*}
\lambda=\frac{\ln (I(t) / I(0))}{t} . \tag{16}
\end{equation*}
$$

We use the fact that the vaccine (post-booster) has proven to be $90 \%$ effective in preventing clinical symptoms of CDV (Abdelmagid et al., 2004). From data from a shelter disease outbreak, we estimate an intrinsic growth rate for susceptible dogs $\left(\lambda_{s}\right)$ and for vaccinated dogs $\left(\lambda_{v}\right)$ given values of $\lambda_{s}=.1620$ and $\lambda_{v}=.0254$.

The number of infected dogs in the early phases follows an exponential growth trend; we exploit this characteristic to help determine the disease transmission rates for both susceptible and vaccinated dogs as $\beta N-\gamma=\lambda$ where $\gamma$ is the disease recovery rate, and $N$ is the total population (Keeling \& Rohani, 2008).

The average recovery rate found in the literature was 2 weeks, which gives us $\gamma=\frac{1}{14}$ (Belsare \& Gompper, 2014; Greene \& Appel, 2006). Knowing both the population number $(N)$ and the intrinsic growth rate $(\lambda)$, as well as using a recovery rate of 14 days, a disease transmission rate of $.0053\left(\beta_{s}\right)$ for susceptible dogs and $.0022\left(\beta_{v}\right)$ for vaccinated dogs was calculated.

## Disease progression rate

The disease progression rate, or the rate at which exposed dogs move from exposed to the infected class, $\alpha$, is determined through the latent period of CDV. The latent period of

Table 1. This table shows the estimated parameters for the shelter model.

| Parameter | Estimate (dogs/day) | Units |
| :---: | :---: | :---: |
| Daily drop-off rate, $b$ | $\frac{485}{31}$ | $\frac{\text { dogs }}{\text { time }}$ |
| Vaccination rate, $\delta$ | 31 | time |
| Susceptible adoption rate, $a_{\text {S }}$ | . 0504 | time |
| Vaccinated adoption rate, $a_{V}$ | . 0168 | $\frac{\text { tipe }}{\text { time }}$ |
| Exposed adoption rate, $a_{E}$ | . 0336 | $\frac{1}{\text { time }}$ |
| Susceptible disease transmission rate, $\beta_{S}$ | . 0053 | $\frac{1}{\text { dogs } * \text { time }}$ |
| Vaccinated disease transmission rate, $\beta_{V}$ | . 0022 | $\frac{1}{\text { dogs } * \text { time }}$ |
| Disease progression rate, $\alpha$ | $\frac{1}{7}$ | $\frac{1}{\text { time }}$ |
| Disease death rate, $d$ | 2 | $\frac{1}{\text { time }}$ |

Table 2. The elasticity values of $R_{0}$ in regard to various parameters demonstrate the relative influence of the parameters.

| Parameter | Elasticity of $R_{0}$ | Value |
| :--- | :---: | :---: |
| Vaccinated disease transmission rate, $\beta_{V}$ | $\frac{\delta \beta_{V}}{a_{V} \beta_{S}+\delta \beta_{V}}$ | .47 |
| Susceptible disease transmission rate, $\beta_{S}$ | $\frac{\beta_{S}}{\beta_{S}+\frac{\delta}{a_{V}} \beta_{V}}$ | .53 |
| Susceptible adoption Rate, $a_{S}$ | $\frac{-a_{S}}{\delta+a_{S}}$ | -.59 |
| Death rate, $d$ | -1 | -1 |

CDV varies from dog to dog based on the dog's relative immune system strength, but it has an average period of 7 days (Greene \& Appel, 2006). Therefore, we estimate a disease progression rate of $\frac{1}{7}$.

## Disease death rate

We determined that at this shelter, where they euthanize upon the dog showing clinical signs and a positive test result, a death rate, $d$, of two would adequately represent the euthanasia policy and speed of test results. This death rate results in approximately $85 \%$ of infected dogs being euthanized daily. Our estimated parameters for the shelter model, equations (12)-(15), are shown in Table 1.

## 3. Results

### 3.1. Shelter application results

Using our estimated parameters for CDV, the local shelter we modelled had a basic reproductive ratio of $R_{0}=.73$. Since it is below 1 , a disease outbreak would not persist at the shelter. Upon introduction of one exposed animal into the shelter, our simulation results show that no disease outbreak occurs (Figure 4). If the shelter had delayed euthanasia policies, either due to a slow return of the test results or lack of funding to conduct the tests, upon introduction of one exposed animal into this shelter, a disease would run rampant as shown in Figure 5. This simulation used a euthanasia value of $d=\frac{1}{7}$, which resulted in a basic reproductive ratio of $R_{0}=10.2$.



Figure 4. The population dynamics with the introduction of one exposed dog, $E(0)=1$. With the current prompt euthanasia policy upon infection $(d=2)$ at this shelter, the introduction of one exposed animal does not cause a shelter-wide outbreak.


Figure 5. If a shelter fails to euthanize infected animals promptly ( $d=1 / 7$ ), the introduction of one exposed animal results in a shelter-wide outbreak.

### 3.1.1. Sensitivity and elasticity analysis for local shelter

The elasticity of R 0 with respect to p is given by

$$
\begin{equation*}
\frac{\partial R_{0}}{\partial p} \frac{p}{R_{0}} \tag{17}
\end{equation*}
$$

Elasticity measures how the relative increase in one input affects an output. If the elasticity value is negative, the output decreases when the input is increased, while when the value is positive, an increased input value results in the output increasing as well. Elasticity provides us with a means to evaluate various control methods as different parameters in our model represent various methods. For instance, our transmission rates represent both the vaccine

Table 3. A summary of the sensitivity analysis performed on the parameters found in the model, with Min and Max giving the range for each parameters.

| Parameter | Min | Max | PRCC |
| :--- | :---: | :---: | ---: |
| $b$ | 10 | 20 | .38 |
| $\beta_{S}$ | .003 | .008 | .93 |
| $\beta_{V}$ | .001 | .003 | .16 |
| $\delta$ | .018 | .054 | .04 |
| $a_{s}$ | .025 | .076 | -.26 |
| $a_{V}$ | .025 | .008 | -.12 |
| $a_{e}$ | .017 | .05 | -.24 |
| $\alpha$ | .072 | .214 | .05 |
| $d$ | 1 | 3 | -.94 |

efficiency as well as the probability of interactions with other dogs and therefore, if this parameter has a large effect on the shelter's $R_{0}$, we know a good control option is to limit interactions among the dogs or to improve the vaccine. Furthermore, the elasticity of $R_{0}$ with respect to the adoption rates provides a way to evaluate quick turnover as a control method. Lastly, the death rate and its effect on $R_{0}$ can be used to evaluate the importance of euthanasia as it pertains to the disease's persistence in the shelter. We therefore calculated the elasticity of $R_{0}$, the basic reproductive number, in the local shelter with reference to these various parameters as seen in Table 3.

Since a goal of this project was to analyse ways to limit an outbreak of CDV, we performed a sensitivity analysis on the parameters in the model. We used the local shelter model with initial conditions $S_{0}=147, E_{0}=4, V_{0}=83$ and $I_{0}=8$ to assess how changes in each parameter value impact the total number of exposures. A Latin Hypercube Sampling technique was used to sample a reasonable range of each parameter under the assumption that they are uniformly distributed (Marino, Hogue, Ray, \& Kirschner, 2008). We first verified that the total number of new exposures was monotone with respect to each input parameter. We then calculated the influence of each parameter on the system using partial rank correlation coefficients (PRCC). A PRCC with absolute value near 1 and a p -value less than .01 indicates that the outcome is significantly influenced by, and sensitive to, changes in the given parameter (Marino, Hogue, Ray, \& Kirschner, 2008). The sign of the PRCC shows whether the parameter positively or negatively impacts the outcome of interest. Table 3 shows the range and resulting PRCC values for each parameter in the model. The two parameters with significant PRCC values are $d$ and $\beta_{S}(p<.01)$.

### 3.2. Epidemiological model simulations

The model structured from the local shelter could be expanded to an epidemiological model to investigate a disease outbreak in a shelter that had no disease control methods (no euthanasia of infected dogs). Therefore, the same parameter values calculated for the local model were used. See Table 4 for a complete list of parameters used in the model. It was assumed that the transmission rate between susceptible and vaccinated dogs, with chronically infected dogs was half the rate of with acutely infected dogs. The remaining parameter values were estimated using data from Krakowka and Koestner (1976) and were verified by running a disease simulation in MATLAB where the total number of dogs

Table 4. This table shows the estimated parameters for the epidemiological model.

| Parameter | Estimate(dogs/day) | Units |
| :---: | :---: | :---: |
| Daily drop-off rate, $b$ | $\frac{485}{31}$ | $\frac{\text { dogs }}{\text { time }}$ |
| Vaccination rate, $\delta$ | $\frac{1}{28}$ | tipe |
| Susceptible adoption rate, $a_{S}$ | . 0504 | $\frac{\text { time }}{\text { time }}$ |
| Vaccinated adoption rate, $a_{V}$ | . 0168 | time |
| Exposed adoption rate, $a_{E}$ | . 0336 | $\frac{1}{\text { time }}$ |
| Recovered adoption rate, $a_{R}$ | . 0336 | time |
| Susceptible-acute disease transmission rate, $\beta_{1}$ | . 0053 | dogs*time |
| Susceptible-chronic disease transmission rate, $\beta_{2}$ | . 00265 | $\frac{1}{\text { dogs*time }}$ |
| Vaccinated-acute disease transmission rate, $\beta_{3}$ | . 0022 | $\frac{1}{\text { dogs*time }}$ |
| Vaccinated-chronic disease transmission rate, $\beta_{4}$ | . 0022 | $\frac{1}{\text { dogs*time }}$ |
| Disease progression rate (exposed to acute), $\alpha_{1}$ | $\frac{1}{7}$ | $\frac{1}{\text { time }}$ |
| Disease progression rate (acutely to chronically infected), $\alpha_{2}$ | . 0067 | $\frac{1}{\text { time }}$ |
| Disease death rate from acutely infected, $d_{1}$ | . 0308 | $\frac{1}{\text { time }}$ |
| Disease death rate from chronically infected, $d_{2}$ | $\frac{1}{70}$ | time |
| Recovery rate from acutely infected, $\gamma_{1}$ | . 0402 |  |
| Recovery rate from chronically infected, $\gamma_{2}$ | 0 | $\frac{1}{\text { time }}$ |



Figure 6. Results of an exposed individual being introduced into a shelter population which proceeds with adoptions and drop-offs and does not euthanize infected animals.
dies from CDV be approximately equal to the number that recovered in order to match observed outcomes.

Using these parameter values, two scenarios were investigated.

- Natural disease progression with adoptions and drop-offs and no euthanasia (Figure 6).
- Disease progression with adoptions, drop-offs and euthanasia of acutely infected dogs (Figure 7).

Looking at these two scenarios, in which the only difference was the euthanasia policy of the shelter, a disease outbreak can be seen in the shelter with a poor euthanasia policy


Figure 7. Results of an exposed individual being introduced into a shelter population which proceeds with adoptions and drop-offs but does euthanize infected animals.
(Figure 6), while not in the shelter that euthanizes (Figure 7). In the second scenario, or the one in which the shelter euthanized acutely infected dogs, the death rate was no longer just the death due to disease but included the death due to euthanasia (euthanasia rate of 1 ). As the euthanasia rate increases, our basic reproductive ratio falls drastically as scenario one had a basic reproductive number of 22.4 while the second scenario had an $R_{0}$ of 1.6 .

## 4. Discussion

The local shelter on which we based our model is exemplary in many ways, including the vaccination policies as well as limiting interactions between dogs. Due to these policies, the corresponding shelter model has a basic reproductive ratio of less than 1.

Through the elasticity analysis, it was found that both the euthanasia and adoption rates have an inverse relationship with the disease's persistence; as the rates increase, the likelihood of an outbreak decreases. These two control methods, euthanasia representing preventative control and the adoption rates, corrective control, have similar effects on $R_{0}$. Therefore, if a shelter wishes to prepare for an outbreak, increasing adoption rates (e.g. through lowering adoption fees) would reduce the number of susceptibles and prevent unnecessary loss of life. Being prepared to promptly euthanize any infected dogs has the same negating effects on the chance of disease spread. The effects of the transmission rates on $R_{0}$ indicate that in stemming a disease outbreak, limiting contact between individual dogs is crucial. Moreover, the larger elasticity value of $\beta_{S}$ in Table 3 demonstrates that protecting susceptible dogs should be a priority to shelter veterinarian wishing to reduce an outbreak of CDV.

The results of the PRCC analysis indicate that the number of new exposures is most sensitive to changes in the transmission rate between new dogs and infected dogs $\left(\beta_{s}\right)$ and rate of euthanasia of infected dogs $(d)$. Since $\beta_{s}$ is the rate at which new exposures occur in the susceptible class, it is not surprising that it has a positive sign and a significant influence on the total number of new exposures. While vaccinated dogs are also susceptible to infection, their transmission rate is lower and there are far fewer vaccinated individuals, which is why the rates governing this class have low PRCC values. Similarly, since $d$ controls the number of infected individuals present in the system, it makes sense that this parameter has a negative sign and that it highly influences the number of new exposures.

These results show that the specific transmission rate of each new strain of CDV produces significant changes in dynamics. They also convey that obtaining rapid test results and removing infected individuals can significantly limit outbreak.

An example of how the rate of euthanasia impacts the model of a local shelter can be seen in Figures 4 and 5. A value of $d=2$ prevents the spread of CDV in the shelter, whereas $d=\frac{1}{7}$ results in a severe outbreak. The difference between the two values could be as simple as a shelter having access to a PCR machine and thus rapid test results compared with a shelter whose testing is delayed.

These results from the elasticity and PRCC analysis both confirm what was shown in the epidemiological simulations; promptly euthanizing acutely infected dogs works as an effective control in disease management. This, therefore provides evidence that a shelter should be very perceptive to potentially infected dogs and euthanize promptly when an infected dog is found.

While our model does allow us to evaluate the effects of various control methods and investigate disease spread situations, it does have limitations. In the current formulation of our model, including isolation as a preventative measure is not feasible. Since it is the most likely form of containment in a shelter, it is something we hope a future model will be extended to encompass. This model also does not incorporate the different rooms that dogs are housed in, but instead views the shelter as a homogeneous mix of dogs.

## 5. Conclusions and future work

In a shelter's aspirations to reduce the likelihood of an outbreak, the local shelter modelled in this paper may serve as an example of actions to take. The primary means of prevention is to limit the interactions between dogs. However, this may not be a method of prevention a shelter veterinarian wishes to partake in as allowing the dogs to interact with one another increases the standard of living for the dogs.

Although it was found that the interactions between infected and non-infected dogs greatly influence whether a shelter-wide outbreak would occur, it was also found that reducing the number of susceptibles in the shelter or promptly euthanizing dogs can diminish the effects of an introduced exposed dog into a susceptible shelter population. Euthanizing an infected dog promptly is a financial burden, but it may be worthwhile in preventing a CDV outbreak. The local shelter we modelled was able to test animals for CDV quickly and inexpensively, due to having access to a PCR machine. This machine is an upfront cost, but may help prevent shelter-wide depopulation and save funds in the long term.

Until a more effective vaccine has been created and made available, the local shelter model in this paper provides suggested preventative and corrective control methods that a shelter can enact in order to lower the chance of a shelter-wide outbreak. Lastly, the results discussed in this paper may or may not be applicable to the newest strain of CDV due to the lack of data available.

In the future, it is important to investigate the effect of vaccination policy on the disease spread, as well as see how a disease spreads and persists in a shelter with no vaccination as there is a new strain of CDV currently circulating, that has been known to evade vaccine protection. Furthermore, we plan to extend our model to incorporate different age classes to include the various age-related susceptibilities to CDV.

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## Disclosure statement

No potential conflict of interest was reported by the author.

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