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EDUCATION ARTICLE

Modelling and simulation: helping students acquire this skill using a Stock and Flow approach with MathBench

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Computational and modelling skills are vital to most fields of biological research, yet traditional biology majors have no or little opportunity to develop these skills during their undergraduate education. We describe an approach, which can address this issue by a synergy of online resources called MathBench modules and Stock and Flow modelling. Using a step-by-step method starting with a MathBench 'bootcamp', we were able to achieve a significant gain in quantitative skills of students with no previous experience with model building. At the end of the course, the students were able to construct and analyse complex models and gained confidence in mathematical skills.

Keywords: course description; mathematical biology; MathBench; Stock and Flow model

1. Introduction

Both Vision and Change (American Association for the Advancement of Science, 2011) and Bio 2010 (National Academies Press, 2003) strongly recommended developing a skillset for undergraduate biology majors that not only allows students to understand mathematical models, but also trains them to construct models. Modelling is one of the cornerstones of establishing a strong foundation in mathematics and information sciences to prepare students for research that is increasingly interdisciplinary in nature (Bialek & Botstein, 2004; Gross, 1994; Karsai & Knisley, 2009). A better mathematical and computational foundation is also very important for the students who will enter the workforce, where interpreting quantitative information (Ewing et al., 2003) and making policy decisions based on quantitative predictions are becoming a desirable skill.

Although mathematics has had a very important influence on biology (Jungck, 1997; May, 2004), bridging the disciplines of mathematics and biology has been problematic (Karsai, Knisley, Knisley, Yampolsky, & Godbole, 2011). In comparison, building interdisciplinary bridges between biology and chemistry or biology and physics has been less difficult because biology shares solid roots and a common language with chemistry and physics. However, there are profound differences between biology and math in motivations and epistemological approaches (Karsai & Kampis, 2010). Karsai and Kampis (2010) argued that simply introducing more mathematics into biology education will not provide a solution.

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To understand the role of mathematics in biology, students need to understand the science before they can make sense of using mathematics to deepen their understanding of the science.

New, active pedagogies such as inquiry- and case-based learning increase student engagement, content retention and knowledge transfer to new situations (Hake, 1998; Kitchen, Bell, Reeve, Sudweeks, & Bradshaw, 2003; Trempy, Skinner, & Siebold, 2002; Udovic, Morris, Dickman, Poslethwait, & Wetherwax, 2002). That science is best learned by doing research is an idea that has been around for decades (Roth, 1995), as has the emphasis on the importance of undergraduate research (McComas, 1998), but it seems that successful approaches to educate and mentor biology undergraduates still need to be developed (Karsai et al., 2011). For example, traditional introductory-level biology textbooks generally do not support these kinds of pedagogies. Moore et al. (2013) found that the current textbooks serve to reinforce rote memorization rather than problem solving, and they in fact de-emphasize quantitative thinking. Instead of a coherent set of unifying principles, biology students will reach upper level courses with an amorphous collection of 'facts'. Pedagogically, there are several approaches that could improve learning transfer, such as improving student's metacognitive skills (Davidson & Sternberg, 1998), inductive and deductive reasoning ability (Lawson, 2005) and providing them challenging assignments or projects (Lawson, Banks, & Longvin, 2007). However, the integration of inquiry-based approaches into the curriculum commonly has flaws, and the true nature of inquiry is often forfeited in this process (Karsai & Kampis, 2010). Collecting data and obtaining results (however quantitative) are commonly part of science, but are not science itself. Karsai and Kampis (2010) emphasized that the operative use of the complete scientific method will play a critical role in providing the necessary underpinning for the integration of math and biology at various professional levels.

Modelling can be implemented early in biology education and it is also a good tool to teach the scientific way of thinking and the process of research (Joplin et al., 2013; Karsai & Kampis, 2010; Weisstein, 2011). Many courses treat equations as 'black boxes' into which one merely plugs numbers and calculates answers (Jungck, 2005). To give meanings to equations, it is crucial to construct the models. Students need to know why they must construct models rather than just use them. For practising scientists, modelling helps to clarify both the conceptualization of system structure and the quantitative relationships among system components (Laurenroth, Burke, & Berry, 2003). Mathematical models provide templates where data, mechanisms, simulations and theories are represented in a uniform and transparent fashion that makes it easy for members of interdisciplinary teams to collaborate (Karsai & Kampis, 2010). These skills are even more important to students but traditional teaching practices generally leave out most of modelling and quantitative reasoning (equations) from biology education.

Here we describe an approach where model making was introduced in an upper level course to a student body with different backgrounds with the goal of bridging mathematics and biology, theory and policy-making, as well as fostering an understanding of the scientific method and participating in group projects. These goals were approached with a synergistic combination of several MathBench Biology Modules (covering the scientific method, understanding of science and understanding of mathematics for science, and basic problem solving) and the application of Stock and Flow modelling (conceptualization of system structure, predicting, decision-making and quantitative reasoning).

2. Material and Methods

East Tennessee State University (ETSU) is a regional state university with about 15,000 undergraduate and 2000 graduate students. ETSU recently became a research intensive university, and it is in a transitional phase where new challenges related to research administration need to be solved. The Department of Biological Sciences has 16 tenure track faculty members and 2 full-time instructors. The biology major core consists of a three-semester introductory biology sequence of lectures with labs and a one-semester genetics course where lab is optional. After the core, the students generally take at least nine credits of biology electives. While many of the core labs are inquiry based, the student have limited opportunities to use models. In one of the core courses (Biology III), the students interact with a few ready-made Netlogo simulations examining the effect of changes of some parameters to a specific output (Jones & Laughlin, 2010). However, with the exception of the Systems Ecology course (which is the focus of this paper), students do not have opportunity to construct their own models or immerse themselves in quantitative biology. This insufficiency of preparation in lower level courses generates several challenges (Eager, Pierce, & Barlow, 2014).

In our curriculum at East Tennessee State University, the only biology course where students learn to construct models is Systems Ecology (recently renamed Modelling Biological Systems). Systems Ecology is a three-credit senior-/graduate-level course taught in a computer classroom for a maximum of 20 students. It provides both biological background information for the subject matter and training in Stock and Flow and agent-based modelling techniques. The course requires no mathematical ability beyond calculus I and no biological knowledge beyond introductory biology. All concepts and skills are developed during the course. The course uses Ford's book (Ford, 2010) *Modeling the Environment*, technical papers, problem sheets and papers and links for background information. The goal of the course is to teach students to build models and analyse the predictions of those models. Beyond describing biological phenomena with models, we emphasize using models to explain what-if scenarios for decision-makers (non-biologists and non-modellers). Students also learn to use free model developing tools such as Vensim (Eberlein and Sternbeg, 1992) and Starlogo TNG (http://education.mit.edu/projects/starlogo-tng), which help them develop skills that are transferable to future graduate study and employment.

Many biology students might feel intimidated to take such a quantitative course as an elective, particularly if they have had no positive experience with quantitative approaches before. In practice, this means that the course typically has low enrolment. Because the course is open to both graduate and undergraduate students, and welcomes students of any major who have completed basic biology, the backgrounds of the students are very diverse. To mediate these challenges, we found MathBench Biology Modules (mathbench.umd.edu) extremely useful. MathBench is a freely available integrated online suite of 40+ modules with instructor resources (quizzes, tests and summaries). The modules are self-contained and make possible guided or self-paced learning of quantitative skills and concepts in biology. Instead of a spookbook or cookbook approach, MathBench uses an intuitive approach and includes standardized instruments to assess student quantitative skills. We implemented several MathBench modules to provide a 'bootcamp' for the students to sharpen their quantitative reasoning skills so that they could be successful from the start of the course. While MathBench was designed primarily for first- and second-year students, it can be used very effectively for higher level courses. In this course, MathBench was used as a background skill development tool, followed by background information on the course content areas. The Systems Ecology course was formally assessed during two successive offerings, in Fall 2011 and Fall 2012. Enrolment was 11 in 2011 and 12 in 2012. In each year, the majority of students were advanced undergraduates, but there were two graduate students in 2011 and two in 2012.

2.1. Evaluation and Assessment

In each year, students were asked to complete a pre-test at the beginning of the course (before the first MathBench modules were assigned) and a post-test at the end of the course. The test of quantitative skill was a slightly modified version of a tool developed to assess the efficacy of MathBench Biology Modules (Thompson, Nelson, Marbach-Ad, Keller, & Fagan, 2010). This tool consisted of 18 questions that were designed to measure the acquisition of nine basic quantitative skills. For each skill, one question was intended to measure a basic level of understanding, while the second was more difficult to allow discrimination of more advanced understanding. In addition to the correct answer and three distractors, each question also contained the option 'I don't know how to approach this problem'. Students were instructed to select this answer if they did not know how to solve the problem, rather than guessing at an answer. The pre-test consisted of 23 questions, 18 of which measured quantitative skill as described above. The remaining pre-test questions asked about the students' previous math coursework and their attitudes toward the relationship between math and biology. Specifically, we asked students Likert-style questions probing the extent to which they (1) liked math and (2) felt it important that biologists know math. The post-test consisted of 31 questions. Eighteen of the questions were isoforms of the pre-test questions, with only slight numerical and contextual changes to problems. The remaining questions probed students' attitudes, the degree to which they felt the course had increased their scientific content knowledge and quantitative skills, and which elements of the course contributed to these gains.

All students completed both the pre-test and the post-test, and so were included in subsequent analyses. Changes in quantitative skill were analyzed with a repeated measures analysis of variance, with main effects of previous math course work (algebra, precalculus, calculus I, calculus II or higher), concurrent enrolment in a math course (yes or no), and year of enrollment in the course (2011 or 2012). In addition to analysing the change in raw scores, we also expressed the difference in pre- and post-test scores as a normalized change score. This measure was proposed by Marx and Cummings (2007) and is similar to the long-used normalized gain score (Hake, 1998), but does not have the low pre-test score bias of the normalized gain score and is preferable in situations where some student's scores decrease between pre- and post-test. Normalized change scores can range from -1 to 1. For students whose score increased across the semester, normalized change was calculated by subtracting each student's pre-test score from the post-test score and dividing by the total increase possible, based on the student's pre-test score. For students whose scores decreased across the semester, normalized change was calculated by subtracting each student's pre-test score from the post-test score and dividing by the total decrease possible, based on the student's pre-test score. Individual student change scores were then averaged to provide a mean for the population. There were no significant differences between years, so results are reported combined across years. Attitudinal questions were coded based on whether a student's post-test answer reflected a more positive attitude (e.g. higher levels of agreement with the statement 'I like math' or 'It is important for biologists to know math'), a more negative attitude, or no change from the pre-test. Attitudinal questions were then analyzed using a Wilcoxon signed-rank test.

In an effort to better understand the impact of integrating MathBench modules into the curriculum, we also asked the students to provide feedback on the value of the modules. Specifically, we asked 'What role did MathBench Biology Modules have in the development of your scientific content knowledge and quantitative skills?' Student responses to this question were analysed using an inductive process in which the responses were grouped by common themes (Maykut & Morehouse, 1994).

3. Results and discussion

3.1. Integration of course content with MathBench

The course generally focused on ecological problems, starting with a population ecology primer and the introduction of modelling tools. Simple population growth and migration models (Knisley, Karsai, & Schmickl, 2011) were followed by population interaction models (competition, predation and microparasitic disease) and then a series of 'real problem scenarios' such as irruption of deer at the Kaibab plateau, water management of Mono Lake, the tragedy of Easter Island and flu and smallpox epidemics. The course was designed so that students were able to grow from having minimal skill in modelling to being able to build their own model and analyse it by the end of the semester. The course required a final project, which consisted of developing a model that related to the student's own research (if possible). The final project could be done as an individual, a pair of students or a team, and was presented orally to an audience that included students and invited faculty members.

This progression in skill development required a great deal of growth and understanding from students who typically had no experience with problems of this type. MathBench was used to establish basic quantitative and problem solving skills. Specific MathBench modules were selected both for their focus on specific quantitative skills and their biological content, which resulted in a very good synergy between the content of the modules and the goals of the course.

The first half of the course consisted of basic preparation for the more advanced modelling, and it was vital that all students were able to understand the basic concepts and skills required for the more difficult tasks. The first weeks were especially crucial for helping students that needed some review or to be brought up to speed with the other students in the class. MathBench modules on 'Basic Rules of Probability' and 'Sampling' provided a review of probabilities and basic quantitative thinking. The models called 'Exponential Growth and Decay', 'Bacterial Dynamics' and 'The Mystery of the Missing Housefly' provided the foundation for elementary modelling work. The quizzes available through the MathBench instructor resources were used to monitor students' progression during the semester. The second half of the course built on this foundation and focused on advanced modelling.

3.2. An example for a 'real-world' advanced model

After the students learned the basic concepts of modelling, the course focused on real-world models, which enabled them to gain more advanced skills and a deeper understanding of complex biological problems. The first of such models built by the students was the Mono Lake hydrology model (Ford, 2010). First, the students learned about the lake's hydrology and biology, and the effect of the use of Sierra Gauged Runoff (the most important water source for the lake) by Los Angeles. The history and the data gave the students a timeline and parameters to use for the model. Then we discussed two goals for building a model:

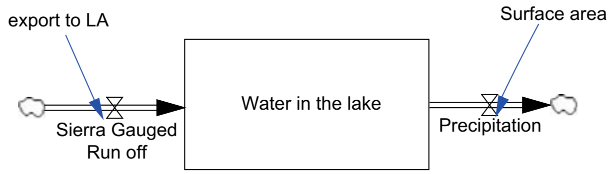

Figure 1. Simple Stock and Flow model of Mono Lake. Rates of inflow and outflow determine the dynamics of water quantity in the lake.

providing a quantitative, accurate description of the past trend and serving as the basis for future management plans of conservation biologists.

Stock and Flow modelling is a very natural approach for this problem, because there is a single stock (the quantity of water in the lake) and in and outflow of water from this lake. Using Vensim, the first step for the student is to draw a simple sketch of the logical relationships of the components of the biological system. The box in the model represents the stock or level where the water is stored (Figure 1). Double arrows represent flow of materials. In case of Mono Lake, water comes in from the Sierra and leaves through evaporation. Variables with single arrows show dependencies in the system. In this case, water entering the lake depends on how much of this water is exported to Los Angeles, and the evaporation rate is dependent on the surface area of the lake (Figure 1).

This sketching phase requires no mathematical knowledge, only logical thinking, but if done correctly the largest part of the model is already accomplished. The next step is implementing the functions into the model. With Stock and Flow modelling, we do not need to use complete equations; the students just need to understand rates. The MathBench modules prepare the students for this independently of their mathematical background. It is easy for students to understand that the amount of water flow into the lake will be the difference between the quantity that comes down from the Sierras (which is assumed to be constant) and the amount that is exported to Los Angeles. This export amount will be an experimental parameter later for decision-making. Precipitation must also be taken into account as some constant (water amount/surface unit), which needs to be multiplied by the actual surface area of the lake. The students also need to fill the lake at $t = 0$ with the amount of water as a starting condition of the model. We explain to the students that the stock actually mathematically integrates the results of the flows, and it means that this is the amount we need to follow and compare with the historical data.

Students will realize that the model does not provide realistic predictions (e.g. after a while they have a negative amount of water in the lake), and step by step they can add more complexity and realism to the model including dynamical surface area, the dependence of evaporation on salinity and so on. Also, from the volume of water they will calculate the elevation of the lake surface, which can be used as a proxy for ecological turning points (Figure 2). It is very useful to translate quantitative data to simple turning points for decision-makers. For example, if the elevation falls below 6363 feet, then the lake's salinity level goes above the critical level that kills even salt tolerant organisms in the lake. With this model, the students can predict the future of Mono Lake, and they also can do experiments by changing the water exported to Los Angeles and trying to save the lake or finding some compromise between lake health and water use of Los Angeles. Students can use the model to explain to decision-makers the quantitative and qualitative consequences of different management practices.

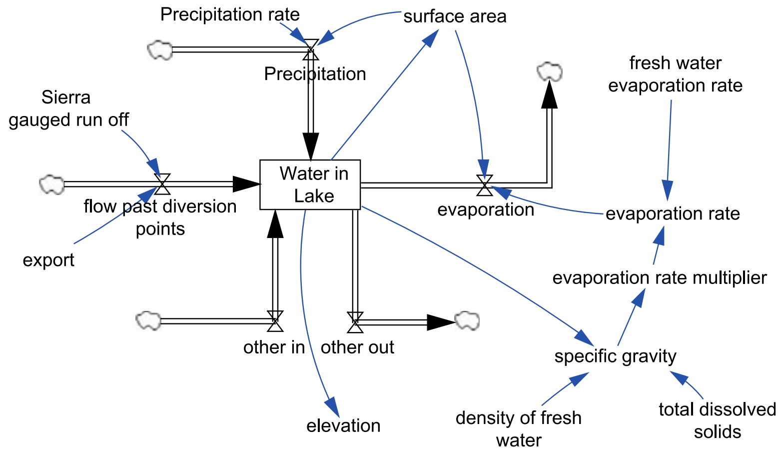

Figure 2. Complex Stock and Flow model of Mono Lake. The box in the model represents the stock or level where the amount of water is followed. Double arrows represent flow of materials. Variables with single arrows show dependencies in the system.

Using Vensim to construct the Mono Lake model is an excellent way to ease students into quantitative modelling. Sketching the visuals (tubes, valves and stocks), which in this case mimics the real situation very closely, makes the modelling a logical effort instead of an exercise of coding or solving differential equations. After the students gain confidence with rates, they are ready to translate their equations into a Vensim model (generally Lotka Volterra prey-predator model) or later translate Vensim models to equations. Their final project requires a model that is at least as complex as the Mono Lake model (Figure 2).

3.3. Evaluation of student learning

When asked about the degree to which the course enhanced their scientific content knowledge, 20 of 23 (87%) students indicated moderate or a great deal of improvement (Figure 3(A)). They reported similar gains in quantitative skill as a result of the course. Sixteen of 23 (69%) students indicated moderate or a great deal of improvement in quantitative skill (Figure 3(B)). These gains were corroborated by the MathBench test of quantitative skill, which showed that scores improved significantly between the pre-test and the post-test (Repeated measures ANOVA, within subjects effect of time: $F = 17.20$, df $= 1, 18$, $p = 0.0006$, Figure 4). This corresponded to a normalized change score of about 37%. This gain was substantially larger than that observed in previous analyses of the effect of MathBench on students in first-year introductory biology (with normalized change scores of about 25%) and second-year organismal biology and genetics courses (normalized change scores of about 10%) (Thompson et al., 2013).

The probability of a student answering ‘I don’t know how to approach this question’ decreased significantly between pre-test and post-test (Repeated measures ANOVA, within subjects effect of time: $F = 9.54$, df $= 1, 18$, $p = 0.0063$). This indicates a greater willingness of students to attempt to solve quantitative problems, whether or not they ultimately arrive at the correct answer. We found no significant effect of year, students’ math background, or whether students were concurrently enrolled in a math course on either gains in quantitative skill or probability of answering ‘I don’t know’.

Table 1. Categorization by emergent themes for student responses to the question ‘What role did MathBench Biology Modules have in the development of your scientific content knowledge and quantitative skills?’ Responses sum to more than 100% because an individual response could be classified into multiple thematic categories.

Theme	Number (%) of responses	Representative quotes
Relevant to real life or biological sciences	11 (48%)	• MathBench modules were helpful in presenting simplified but relevant situations in which math skills can be applied logically. • [MathBench] taught me new methods of quantitative analysis of models and systems that will be useful inside and outside of biology. • MathBench presents statistics in an easily understandable way that is also biologically relevant.
Refreshed math skills	7 (30%)	• [MathBench] refreshed my memory on a good number of computational skills. • The modules benefited me most simply by giving access to many examples and practice questions that help to refresh and round out my current understanding of mathematics.
Interactive or engaging nature of modules	4 (17%)	• [MathBench] helped me improve [my quantitative] skills by incorporating extra practice and tutorials with explanations.
Reinforcement of lecture content	4 (17%)	• [MathBench] helped to reinforce what we learned in the lecture because it gave practical applications for the material.
Systematic and progressive nature of modules	3 (13%)	• [MathBench modules] individualized each individual portion of the content, building upon prior knowledge.
Enhanced critical thinking skills	3 (13%)	• MathBench both increased and challenged my scientific knowledge and quantitative skills. It required me to use both my mathematical background and critical thinking to solve biological issues and examples.

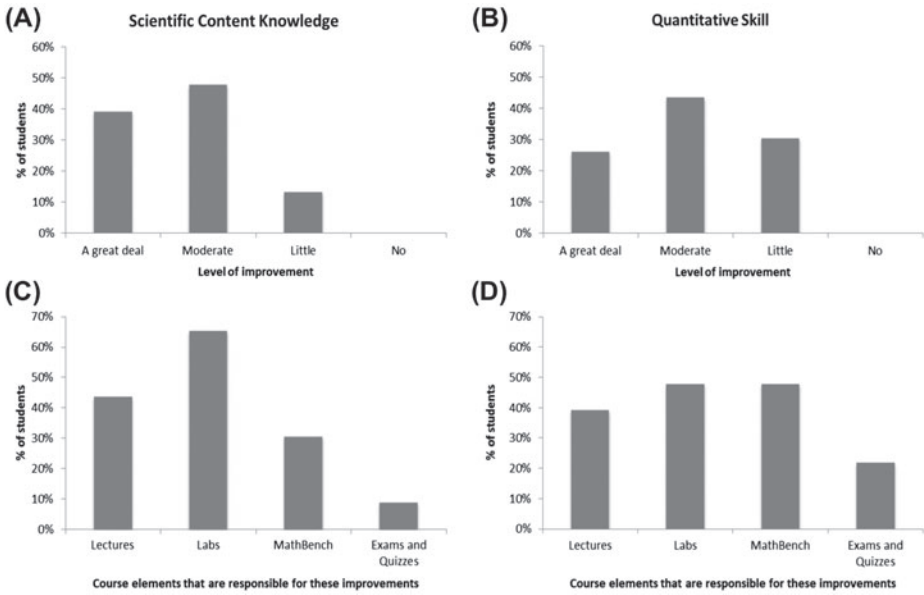

Figure 3. Student assessment of the degree to which their scientific content knowledge (A) and quantitative skills (B) improved during the Systems Ecology course at East Tennessee State University and to which course elements they ascribed this gain (C and D). Course elements responsible for improvements sum to more than 100% because students could select more than one option.

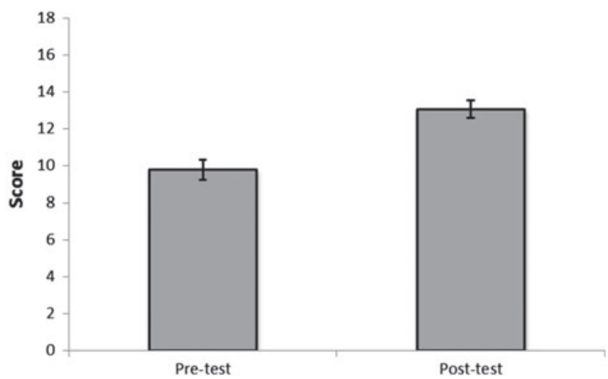

Figure 4. Change in scores on a pre- and post-test of quantitative skill administered at the beginning and end of the systems ecology course at East Tennessee State University.

Students indicated that labs were the element of the course that were most helpful in improving their scientific content knowledge, with course lectures ranked second (Figure 3(C)). With respect to quantitative skills, students cited labs and MathBench tutorials as being equally important for their perceived gains, with lectures being a close third (Figure 3(D)). This fairly even distribution across three course elements is indicative of the well-integrated nature of the course design in helping students hone their quantitative skills.

There was no change from pre- to post-test in the degree to which students felt math was important to biologists or the degree to which they liked math, although this may be due to students' very high level of appreciation for the link between math and biology from the start of the course. All students agreed or strongly agreed with the statement that 'It is important for biologists to know math'.

Students' open-ended responses indicated that the main contribution of MathBench to their learning was the way in which it imbedded quantitative material in biological contexts (Table 1). A significant fraction of students indicated that MathBench helped them refresh their existing quantitative skills. This is not surprising, given that MathBench content largely focuses on the application of algebra and pre-calculus mathematics, while biology majors have typically completed more advanced math coursework such as calculus. The generally positive feedback on MathBench's ability to refresh their existing skills and illustrate their application to biological contexts shows the need for supplementing traditional mathematical training; math courses alone are insufficient for many biology students to feel confident in applying mathematical approaches to biological problems. Smaller fractions of students indicated that they value the interactive nature of the MathBench modules and the way they progressed from intuitive explanations to more sophisticated mathematical approaches. This is consistent with the fundamental design elements that underpin MathBench (Nelson, Marbach-Ad, Fagan, Thompson, & Shields, 2009).

4. Conclusions

Biological processes provide a compelling context for teaching mathematical modelling. For example, compartmental models of migratory dynamics are a very attractive way of introducing derivatives, integrals and the fundamental theorem of calculus (Knisley et al., 2011). However, because mathematical biology and mathematical modelling courses are most often taught by mathematicians within mathematics departments, these courses are frequently overlooked by biology majors. One solution is a course co-taught by faculty members from mathematics and biology that is cross listed within both departments. Karsai co-taught Systems Ecology several times with Jeff Knisley from the Department of Mathematics and Statistics (Karsai et al., 2011). Although this can be an ideal scenario, it presents logistical challenges. An alternative solution, using MathBench modules to provide some of the quantitative content, worked well due to the large selection of modules and the robust instructor support resources. This created a strong synergy between the disciplinary and quantitative content. The students were able to achieve significant gains both in quantitative skills and in biology content knowledge as they progressed from having no modelling ability to being able to build their own model related to their own research interest.

The gains achieved by Systems Ecology students on the MathBench test of quantitative skill are larger than those reported previously for introductory-level courses (Thompson et al., 2010, 2013), which is somewhat unexpected because the modules were designed specifically to help shore up basic skills necessary for success in first- and second-year biology courses. MathBench is typically used as a self-contained platform for students to practice their quantitative skills, and in many cases these skills are not reinforced through subsequent course content. The Systems Ecology course, in contrast, continually engaged students in the application and refinement of these skills. The deep integration of MathBench content into the Systems Ecology curriculum represents a key difference in the implementation of MathBench and undoubtedly contributed to the large learning gains observed.

Modelling contributes to a deeper understanding of biological systems, especially with regard to the nature of the relationships between components, comparisons of different predictions of alternative conceptualizations and identification of general principles across different scientific fields (Turner, 2003). This promotes mathematical and equation literacy. The Systems Ecology course at ETSU provides students with these general outcomes, and in addition helps students learn to connect knowledge within and between disciplines, understand the importance of precise statements and initial conditions for quantitative work, communicate with audience and decision-makers using models as tools, recognize the importance of informed quantitative decisions, and balance biological complexity with computational feasibility.

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