EDITORIAL

Should N95 Respirators be Recommended for the General Public: A Mathematical Explanation

Ashok Srinivasan,[a] Anuj Mubayi,[b,c,d,e] Olcay Akman[d]

[a] Department of Computer Science, University of West Florida, Pensacola; [b] PRECISIONheor, Los Angeles; [c] College of Health Solutions, Arizona State University, Tempe; [d] Department of Mathematics, Illinois State University, Normal; [e] Prevention Research Center, Berkeley

ABSTRACT

Public health advisories recommend against the use of the N95 respirator by the general public in the current COVID-19 pandemic. These advisories are primarily motivated by the collective goal of reducing the reproduction number to below one. However, cultural factors may dissuade the public from adopting recommendations from models optimized for the collective good. This article presents a discussion of mathematical issues that ought to guide an advisory from an individualistic perspective. In particular, we argue that the public health advisory does not appear justified if one considers non-linearity in the dose-response relationship and heterogeneity in infection load in the context of the COVID-19 pandemic. The N95 respirator promises far greater effectiveness than homemade or surgical masks. However, due to a considerable variation in masks' brands and efficiencies, the public should look into the specific details of each available mask option.

1 Introduction

The SARS-COV-2 coronavirus, responsible for the COVID-19 disease, spreads mainly from person to person through respiratory droplets and aerosols. Successful transmission of infection is typically through close contact of a susceptible person with an infective person. Coronavirus-infected individuals shed viruses when they talk, breathe, or cough, and an unprotected susceptible person in close vicinity can breathe in these airborne viruses or droplets and become infected. Therefore, it is suggested that everyone in public places should cover their noses and mouths, especially until vaccines or effective treatments are discovered and widely deployed.

The U.S. Centers for Disease Control and Prevention (CDC), World Health Organization (WHO), and local public health officials have been recommending the use of face masks in public places to control the spread of the novel coronavirus pathogens (WHO, 2020). Recommendation to the general public includes wearing different types of face masks such as handmade fabric masks and disposable medical masks (see Figure 1 for some types). However, they advocate against N95 respirators for the general public, advising them for use only in health care settings (FDA, 2020).

Though, a crucial question is, "How effective is each of these face coverings and how scientifically justified are the public health advisories on this matter?"

1.1 What are N95 Respirators?

N95 respirators potentially promise the most protection against respiratory diseases, including the novel coronavirus. N95s protect the person wearing the mask from virus-laden particles in the air breathed in because their filtering medium is required to filter out at least 95% of particles in NIOSH tests. According to the CDC, there are even more effective respirators available than N95 respirators, such as the N99 (99% filtration), N100 (99.97% filtration), R95 (95% filtration, and partially resistant to oil), and P95, P99 and P100 (95%, 99%, and 99.97% filtration respectively, and strongly oil resistant) (CDC, 2020). However, leakage through the face-seal is the primary limitation at these filtration efficiency levels; so, the higher filtration efficiencies would not necessarily yield improved protection against viruses (Grinshpun et al., 2009). While studies still need to thoroughly evaluate

CONTACT Ashok Srinivasan asrinivasan@uwf.edu

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Figure 1: Difference between regular masks and the N95 respirator.

the role of other different types of masks (fabric and disposable) on the level of acquisition of coronavirus related respiratory droplets and aerosols, there are existing studies that examine their effectiveness with particles of the sizes that carry these viruses.

1.2　Current Public Health Recommendations on Mask Use

Public health advisories recommend against N95 respirator use by the general public (WHO, 2020; FDA, 2020). On the other hand, they recommend them for health care providers. One might wonder why viruses would be hindered by masks on doctors and nurses but not on others.

In this study, we use a systematic mathematical approach to scientifically evaluate reasons in favor of or against this public health advice.

One cause for the disagreement is that an individual's health goals may differ from those of public health agencies (similar to partial equilibrium versus general equilibrium concepts in health economics). For example, the public health goal generally is to reduce the effective reproduction number of the COVID-19 pandemic below one. On the other hand, individuals' goals are to keep themselves and their families safe and healthy. While these two goals are compatible, they are not identical, as we will see later.

In addition, we believe that public health agencies have made the following errors in their advisories about N95. They (i) overlooked rich engineering literature showing the effectiveness of N95 masks, (ii) did not adequately account for heterogeneity and non-linearity in infectioun load, and (iii) misunderstood how to apply science to decision making.

In the rest of this article, we first examine certain mathematical aspects related to modeling infection spread in specific locations. We use these to explain why N95 or equivalent mask use by the general public could be effective. We then examine *the benefits of the use of masks during air travel* as a specific example. However, a similar analysis can also be applied to other venues where one may be at *risk of high exposure* to the SARS-COV-2 virus. We deliberately did not use the expression *high risk of exposure*. We will explain the distinction between the two terms later in this article.

2　The mathematics of masks

A typical individual has to ingest a certain number of virus particles to become infected. This *infective dose* is estimated to be a few hundred particles in SARS and the novel coronavirus. In contrast, it is estimated to be around thousands of particles in the case of MERS. The amount of viruses ingested could also impact the severity of the disease (Van Damme et al., 2020). In any case, there is much individual variation in susceptibility to a disease, and so a *dose-response* relationship defines the likelihood of becoming infected for a specific exposure dosage.

One also needs to consider the dosage that a susceptible person is likely to be exposed to on encountering an infective person. Severe symptomatic patients of influenza or SARS are more likely to shed large amounts of viruses and thus transmit the infection to others. However, asymptomatic patients of new coronavirus can also have a high viral load. In addition, there is a considerable variation in the viral load of different infective persons, varying by a factor of tens of millions (Jacot et al.). In COVID-19, a higher dose may be the cause for some young individuals falling severely ill despite no prior health conditions.

It is essential to understand the dose-infection relationship to obtain insight into the fundamental mechanisms responsible for successfully transmitting infections. There is much literature that describes the underlying relationship between pathogen dose from infectious persons and disease in a susceptible person (Lunn et al., 2019; Conlan et al., 2011). Non-linearities hidden in this dose-response relationship, which determines the likelihood of transmission of infection, is believed to be one of the critical factors governing disease transmission dynamics.

What are some challenges in identifying a dose-response relationship?

According to the definition, the dose-response function characterizes the relationship between exposure to a specific dose of a pathogen and the probability of successfully developing an infection, where the dose is the number of pathogen particles entering the host according to transmission mechanisms of the disease. In COVID-19, this is primarily from direct contact, although airborne transmission also appears to be a significant route in superspreading events. There are many challenges in modeling a realistic dose-response relationship. For example, what is the difference between frequent low-dose exposures versus a few high-dose exposures towards successful infection transmission? How long should a time gap be between exposures for them to be considered distinct ones? Is the relationship concave or convex at low-dosage? We adopt a popular type of dose-response relationship below. However, our essential insights remain identical for other relationships, although specific numbers would change.

2.1 Dose-response relationship and its link with transmission dynamics

The probability of becoming infected clearly cannot be linear with pathogen dose, because the probability is bounded by 1 while the dose is theoretically unbounded. A dose-response relationship that relates the number of pathogens (dose d) with the probability of infection (response $P(d)$) should satisfy the following conditions (Brouwer et al., 2017).

(i) The probability of infection is zero when there are no pathogens ($P(0) = 0$),

(ii) $P(d)$ is a monotonically increasing function of dose ($P'(d) > 0$), and

(iii) The relationship saturates as dose becomes large ($\lim_{d \to \infty} P(d) = K$, where $K \leq 1$ is constant).

The probability of occurrence of infection, $P(d)$, can be modeled stochastically as a function of the exposure dose d, which can be estimated using data capturing the proportion of a successful infection. This following definition of $P(d)$ considers an assumption that each pathogen particle in the dose has a constant probability r of causing the successful infection response, which is the same for every particle, and that they all act independently of each other (Brouwer et al., 2017). If the value of r is very small, then we can assume that the number of successful particles follows a Poisson distribution. Suppose that the infection response is successful when at least one particle succeeds. Then, the exponential model yields the following probability of successful infection response (Brouwer et al., 2017):

$$P(d) = 1 - exp(-r\,d), \tag{1}$$
$$exp(-r\,d) = 1 - P(d), \tag{2}$$
$$d = -\theta y, \tag{3}$$

where $y = ln(1 - P(d))$ and $\theta = \frac{1}{r}$. The slope of this linear equation can be estimated by fitting against data.

The above model will violate alternate assumptions, such as if we assume that particles don't act independently of each other or that a single particle is insufficient to cause infection. For example, threshold models consider that at least k successful particles are required to cause an infection. In more complex models, we could assume that the probability r may vary among particles or hosts, and hence, it may come from its own distribution. Often the beta distribution is chosen for r leading to a hypergeometric dose-response model. Some of the models that have been used in the literature are exponential, beta-Poisson, Hill function-based (which may have a logistic shape), lognormal, and Weibull models. However, the choice of the model should be tested statistically using experimental data (Ben-Ami et al., 2008; Brouwer et al., 2017; Teunis et al., 2016).

2.2 Use of dose-response relationships in epidemic models

While we focus on infection likelihood for one exposure incident of a person, dose-response relationships are often incorporated into an epidemic model. We briefly summarize this for completeness. In a modified Reed-Frost model, the probability of infection for all infectious individuals in a closed environment, such as a classroom or an airplane, can be given as follows.

$$P(I) = 1 - \exp\left(-R\frac{Iqpt}{Q}\right), \tag{4}$$

where I is the number of infectious individuals, p is the pulmonary ventilation rate of a person (that is, the total volume of air entering the lung per minute), q is the quanta generation rate (i.e., number of infectious airborne particles required to infect the person), t is the exposure time interval, Q is the room ventilation rate with clean air, and R is the fraction of particle penetration of the respirator such as N95 mask (Azimi et al.; Gammaitoni and Nucci, 1997; Sze To and Chao, 2010; Tung and Hu, 2008).
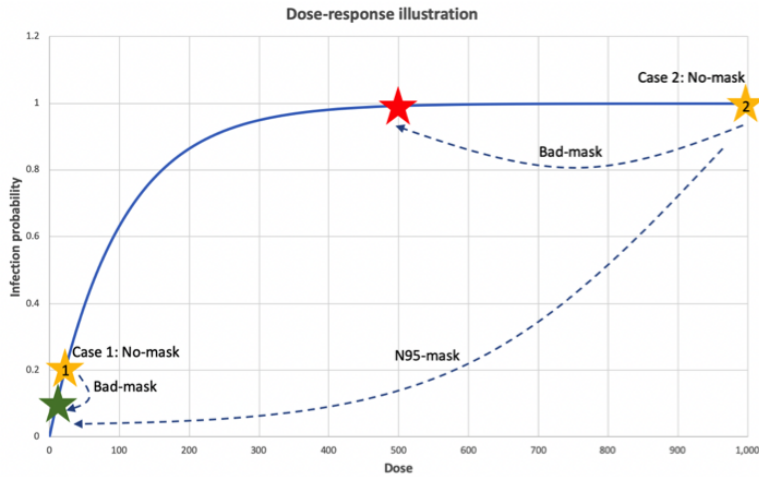
Figure 2: Infection probability as a function of dose (blue line). This schematic illustrates the impact of heterogeneity in infectivity and non-linearity of the dose-response relationship on the effectiveness of masks.

Suppose (S_t, I_t) is a state at time t of an indoor population. Then the modified Reed-Frost will be

$$I_{t+1} = Binomial(S_t, P(I_t)), \tag{5}$$

$$S_{t+1} = S_t - I_{t+1}, \tag{6}$$

where the Binomial distribution can be approximated using a Poisson distribution under certain assumptions. This model describes the evolution of infection in generations ($t \in \{1, 2, \dots, \}$), where each infected individual in a generation independently infects each susceptible individual in the closed population. The basic assumptions made in this model are: (i) a susceptible individual after sufficient contact with an infectious individual will develop the infection and will be infectious to others only for one time period, (ii) in the subsequent time periods after the infectious period, the individual will recover and become permanently immune, and (iii) each individual has a fixed probability of coming into adequate contact with any other specified individual in the population within one time interval, and this probability is the same for every member of the population.

The deterministic limit is found by replacing the random variables with their expectations,

$$I_{t+1} = S_t P(I_t) \approx R \frac{qpt}{Q} S_t I_t, \tag{7}$$

$$S_{t+1} = S_t - I_{t+1}, \tag{8}$$

where $R_0 = R \frac{qpt}{Q}$ is the reproduction number of the model. If $R_0 \leq 1$, then the epidemic dies out, and if however $R_0 > 1$, then the epidemic may take off, infecting a large number of individuals. By manipulating these equations, it can be shown that when $t \to \infty$ and the initial fraction infectives is small, and the rest are susceptible, then S_∞, the fraction avoiding infection during the outbreak, is given by the positive solution to $S_\infty = exp(R_0 * (N - S_\infty))$. This equation may equivalently be expressed in terms of $E_\infty = N - S_\infty$ (where N is the total population size) is called the final size equation.

2.3   Relating mask use to the dose-response relationship

We now examine the impact of relevant mathematical properties of the dose-response relationship on the mask controversy. An exponential dose-response relationship is shown in the blue line in Figure 2. In this example, the dose does not necessarily represent one virus particle. We leave the unit for the dose unspecified. The exponential model is scalable (Brouwer et al., 2017), and the same curve can be used for different diseases by redefining the unit for the dose in terms of the number of virus particles.

Note that exposure to some dose of the virus arises from viruses shed by an infective person while breathing, coughing, sneezing, etc. There is a wide variation in the number of viruses shed by different persons, which can vary by a factor of tens of millions in COVID-19, as mentioned above. So, the dose that a susceptible person is exposed to can vary considerably. The distribution of doses from an infective person impacts the type of masking strategies that would be effective.

For example, consider the following two different situations in which the infection spread probability is 20%.

- In case 1, each infected person exposes others who are "close by" to the same dose. (One may define closeness, for instance, as being within 6 feet if we consider droplet transmission as the primary infection spread mechanism.) This dose would

Table 1: Approximate effectiveness of masks in filtering out particles with viruses (higher is better).

	Inhalation effectiveness		Exhalation effectiveness
	Good fit	Bad fit	Bad fit
N95/FFP2	>99% [a]	97% [a]	60% [b]
Surgical mask	95% [a]	80% [a]	50% [b]
Tea-cloth		60% [b]	15% [b]

[a] Grinshpun et al. (2009),   [b] van der Sande et al. (2008)

correspond to the yellow star #1 in Figure 2, with a dose of around 22 units. If those nearby wore a mask that filtered 50% of the viruses, then the dose received by them would be only 11 units, leading to an effective infection probability of around 10%, shown by the green star in Figure 2. Thus, their infection probability drops roughly by a factor of two.

- Now, consider a situation where eight out of ten infected persons don't infect anyone, but two out of ten infect everyone nearby. On average, if a susceptible person experiences proximity to an infective person, then he/she still has a 20% chance of getting infected, as in the previous case. As an example, we consider that the infective persons expose those close by to doses of 1000 units, marked by the yellow star #2 in Figure 2. In this case, the same mask as above reduced the dose to 500, marked by a red star in Figure 2. Mask use does not substantially mitigate infection risk here; the infective persons still infect those around them with probability close to one. So, the net infection probability, including the infected-but-not-infective persons, remains at 20%. On the other hand, if those nearby wore masks that filtered 99% of the viruses, such as an N95 mask worn correctly, their dose would be only around 10 units, leading to an infection probability of about 9.5%.

Thus, *heterogeneity in infectivity (inequality in the infectivity of people) can make a substantial difference on which masks are effective.*

If we consider COVID-19, it has been estimated that around 10% of infected persons are responsible for 80% of the infections (Endo et al., 2020). The majority of infective persons do not infect others, even when they are in close contact for extended times, such as being in the same household. Such diseases are characterized by a small fraction of infected persons being superspreaders, who spread the virus to many others. Thus, this situation is closer to the second case above than to the first one. In such cases, a few unlucky incidences account for large spreads (excluding household contacts). We need to ask ourselves, "*How does one protect oneself if one is in an unlucky situation with a high dose?*" rather than focusing on protecting ourselves in a typical situation.

In other words, we need to focus on situations incurring a *risk of high exposure* rather than a *high risk of exposure* to some infected person. As the second example illustrates, the efficiency of masks in filtering viruses can make either no significant difference or a substantial difference. We examine results from mask studies next to study this further.

3   Results

3.1   Mask filtration efficiency

Given the controversy about the effectiveness of masks, there has been surprisingly clear evidence available for several years on their ability to filter particles carrying viruses. There are two types of tests (i) inhalation and (ii) exhalation. The first test is useful in understanding if someone would get infected by breathing in particles containing viruses. The second one checks if the mask effectively prevents an infected person from shedding viruses outside the mask.

Viruses are shed within particles of varying sizes, and the effectiveness of masks varies for particles of different sizes. They also vary with activities performed by the wearer, such as no activity, reading, walking, etc. The results below use data for some typical ranges of airborne particles.

The N95 mask has consistently been shown to be very effective for inhalation during various activities and various particle sizes. To measure their effectiveness, we need to consider both the leakage through the mask's filtering material and leakage due to improper fit to the face. The material filters over 99% of particles that could hold viruses. The main limitation is leakage due to an improper fit, which is roughly around 3%. An N95 mask with leakage from poor fit is still much better than other masks with a perfect fit. In exhalation, all masks are less effective than in inhalation, although the N95 (without an exhalation valve) still outperforms others[1].

We can now examine a few high-dose scenarios. In light of Figure 2, if a person would have been exposed to a dose of 500, but is wearing an N95 mask (essentially equivalent to the FFP2 European standard), then that person's effective dose would

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[1]A recent work analyzes a large number of masks using a new testing technique for exhalation. While it, too, shows the N95 as most effective, the numbers differ a little (Fischer et al., 2020). We use data from earlier tests because they rely on established procedures.

Table 2: Infection probability with different masks worn by infective and susceptible persons (the latter in the different columns).

	Susceptible's mask				
	N95		Surgical		Homemade
Infective's mask	Bad fit	Good fit	Bad fit	Good fit	
None	25%	10%	86%	40%	98%
Homemade	17%	6%	72%	27%	92%
Surgical	10%	4%	53%	17%	78%
N95	3%	9%	45%	14%	70%

reduce to around 15, leading to an infection probability of approximately 15%. Much of this is due to poor fit. If one fit the mask well, the dose would be reduced to below 5, with an infection probability of less than 5%. If one wore a surgical mask, on the other hand, the effective dose would drop to around 100, with an infection probability of 63%, while a good fit would reduce infection probability to about 22%. A homemade mask, such as a tea-cloth mask, would lower the dose to 200, with an infection probability of 86%.

Now, let us consider an ambient dose of 1000 without a mask. An N95 mask would reduce infection probability to 10% or 25%, with or without a good fit, respectively. The respective probabilities with a surgical mask would be around 40% and 86%. A homemade mask would lead to an infection probability of approximately 98%, which is virtually useless.

These infection probabilities would reduce if everyone wore a mask. Let us assume that the infected person wears a homemade mask. This scenario reduces the ambient virus shed by only 15%, but it also reduces the distance to which large droplets go. Somewhat arbitrarily, we assume that this reduces the dose by another 25%, to show the qualitative relative impacts. Table 2 gives infection probabilities for a variety of scenarios assuming that the dose would be 1000 if neither the infective nor the susceptible person wore a mask.

These results indicate that (i) wearing an N95 mask helps substantially more than other masks irrespective of what others do and (ii) if everyone wears a good mask, then it benefits everyone significantly (cumulative impact on community transmission). While the N95 mask is in short supply, the roughly equivalent KN95 is widely available. The primary difference is that KN95 restricts leakage, including from a lack of fit, to 8% (3M, 2020). N95 does not have any restriction on this, but in practice, has much less leakage.

3.2   Rationale for advisories against N95 use by the general public

Public health agencies provided the following reasons to discourage mask use initially, and N95 mask use later, by the general public (WHO, 2020; FDA, 2020; Yeung et al., 2020). We critique each argument.

1. People will *not wear or handle them correctly*. This argument was initially applied to all masks, and then later reserved only for the N95 respirators. As the above data shows, this is not a valid argument. **Even with a bad fit, the N95 outperforms other masks.** Furthermore, these results have been available in the scientific literature for years. Even without those studies, this argument would not be based on real science. The way people wear or handle a mask, unlike gravity, is not a law of nature. People can be taught to wear and handle them correctly.

2. Masks would give people a *false sense of security*. The idea is that people might relax on social distancing if they felt secure in a mask. Public health agencies provided no evidence that any vulnerability from this conjectured sense of security is higher than that due to exposure to the virus. Furthermore, this is not a scientific result. People's actions from a sense of security are not immutable laws of nature. While the theoretical case analyses of Table 2 indicate that **it is critical to maintain social distancing even with a good mask,** people can be educated on the importance of social distancing even while wearing a mask.

   Any security measure, such as seat belts in cars, can give a sense of security. Such safety measures are still adopted because their benefits override any conjectured disadvantage from a sense of security. For example, we would not cancel our health insurance policies because that might give us a sense of security that could cause us to indulge in activities that impair health.

3. Another argument is that the *probability of infection is low for an average person, in contrast to health care workers who are at high risk*. By reserving scarce N95 masks for healthcare workers, the public health system may benefit overall. This is a reasonable argument and might be persuasive for some. (Note that the infection probabilities in Table 2 apply to a situation where people are exposed to a superspreader; such exposure is rare, leading to the overall infection probability being small.) This views the argument from a collective perspective.

However, there is an alternative position from an individualistic perspective. One could argue that an N95 mask would lead to tangible risk reduction for individuals and their families. Is it sagacious to sacrifice this clear benefit, trusting mathematical models that don't have a track record of successfully predicting epidemic trends? From this perspective, the use of an N95 or equivalent mask by the general public is advisable. In any case, KN95 masks are available in plenty. So, the dilemma of collective versus individual optimization is less now.

4. Yet another argument is that *N95 respirators are not shown to help the general public.* This argument was first made for all masks and then changed to only the N95 mask. We will discuss this further below.

3.3 Public health studies on masks

Research on the effectiveness of masks can roughly be categorized into two types. One consists of controlled lab tests in which the ability of particles to penetrate the filtration material is tested. This is an engineering-type test, which is very reproducible, with data that is not noisy. Table 1 summarized the results of such tests. It is not the typical type of research carried out by the public health community.

Public health research is often based on observational studies of people or communities with different mask use patterns. The advantage of this type of research is that it can measure the ultimate health outcome—did people get infected?—instead of theoretically modeling infection risk based on a dose-response relationship. The disadvantage of this type of research is that the data is very noisy.

For example, the amount of virus shed by people can vary by a factor of tens of millions, and people's susceptibility can vary by a factor of a hundred, depending on the immune response of both susceptible and infectious individuals and the time since the acquisition of infection by the infectious individual. So, given a pair of infective and susceptible persons, the risk of infection transmission can vary by a factor of a billion. Consequently, when public health research suggests that masks are not very useful, this usually just means that the data was too noisy to come to conclusions.

The difficulty of modeling superspreading compounds this. Superspreading often arises from fat-tailed distributions. In such distributions, rare events can have an enormous impact on the result. Such distributions require much more data for coming to conclusions (Cirillo and Taleb, 2020; Taleb et al.). In fact, for some distribution parameters, it is not even feasible. On the other hand, we know that a susceptible person cannot get infected without the physical mechanism of being exposed to a sufficient quantity of virus. The engineering-type research clearly shows the impact of masks on a necessary condition for infection spread.

There is also a philosophical issue to consider—the difference between generating scientific knowledge and applying science for decision-making (Taleb et al.) and finally implementing it as a policy after political debates. A conservative position (in the sense of being cautious) when coming to conclusions on knowledge is to be sure about it. However, a conservative position concerning decision-making is to avoid potential harm. Wearing a good mask can provide substantial benefits—with a clear mechanism for its action—with much imagination required to identify negative consequences. Besides, the culture of a society adopting a policy impacts its effectiveness. In an individualist society, it would help to have results that support decision making based on individual benefit rather than only on a collective benefit.

4 Example: Application to air travel

We will now examine the risk of COVID-19 spread in air travel and how masks could help. The International Air Transport Association (IATA) has been reluctant to admit to the evidence of COVID-19 transmission in airplanes. However, there have been several studies showing COVID-19 transmission in flights (Eldin et al., 2020; Chen et al., 2020). The number of cases reported may appear small given the scale of the epidemic. However, we need to consider the following caveats.

4.1 Infection risk in air travel

1. The number of cases may be much higher than reported. If people have been exposed to multiple causes of infection, usually the most likely reason is assigned as the cause. For example, 16 people tested positive for COVID-19 on a flight reported in (Eldin et al., 2020). At least three of those had symptoms in a time-frame where it could be due to inflight transmission. However, two of those were not considered inflight transmission because they had contact with an infected person outside the plane too. Assigning all cases to the most likely cause leads to a bias in the estimated frequency of the actual cause.

2. There has been a sharp decline in the number of passengers—far greater than the reduction in capacity—which has led to less crowding. In addition, some airlines have taken steps, such as keeping middle seats empty, which further increases
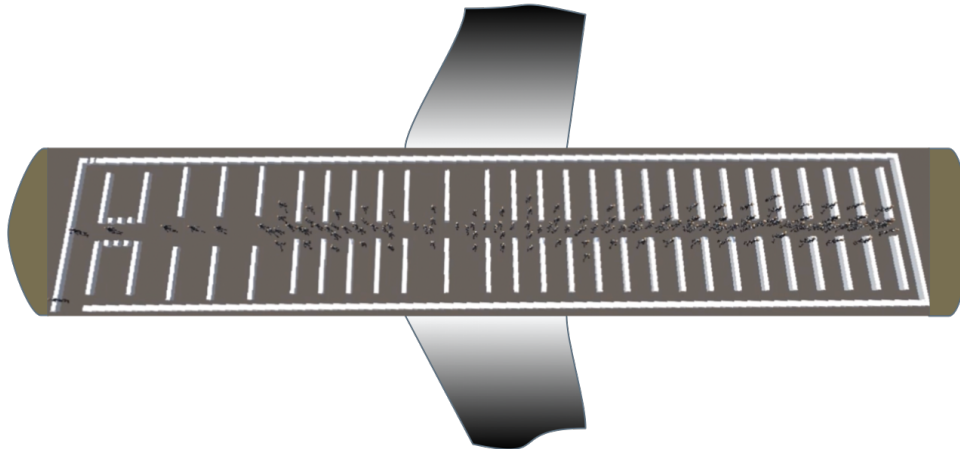
Figure 3: Illustration of passengers deplaning a flight.

social distancing. These have the potential to reduce infection spread risk. Passenger traffic has been growing recently, which could increase the risk.

3. As mentioned earlier, superspreading is ideally modeled as a fat-tailed distribution. Extreme events are rare, but dominate the number of cases. There are normally around 100,000 flights a day. A very low probability event could lead to a massive infection outbreak because the number of flights is so large as to make rare events occur.

   For example, consider SARS. Around 8000 persons were infected with SARS, mainly in Southeast Asia. There were 40 flights with infected persons. Only five flights led to inflight infections, with a total of 37 cases of inflight infections. Twenty-two of these infections arose from one single 3-hour flight (Chang et al., 2003). We can see that most flights did not lead to any infections, those that did almost always led to few infections. But a single flight had an enormous impact on the average number of infections per infected passenger. It may be tempting to discard that flight as an outlier, but that would be a mistake in an infection driven by superspreading. Fat-tailed distributions have the nature that rare events have a substantial impact on the mean. **You see the disaster only after it hits you.** We can see such impacts in other contexts, such as the Diamond cruise ship outbreak, and to some extent in the early outbreak in nursing homes in the state of Washington.

4. With a morbidity count of 8000 people, SARS had close to 40 secondary reported infections in air travel, making infections through air travel 0.5% of the total cases. With COVID-19 infectives often having mild or no symptoms, and with greater air travel now, **normal travel patterns** would yield a much greater fraction. CDC estimates that there are over 20 million infected persons in the USA, and the epidemic is still ongoing. So, we could end up with potentially over 100,000 inflight infections *if no steps are taken*. Of course, effective mask use, and other strategies to increase social distancing, would substantially change the picture.

5. With SARS, the 37 inflight cases led to more than 300 secondary cases. If we had a similar secondary case ratio, we could end up with over a million cases directly or indirectly due to air travel, assuming that other factors remained unchanged.

4.2 Impact of mask use on infection spread in flights

In considering a suitable strategy to be safe during flights, one needs to protect against risk when in proximity to a potential superspreader. In the SARS superspreading case, 8 of 23 passengers in nearby rows were infected (Chang et al., 2003). This is an infection probability of around 35%. In our dose-response relationship in Figure 2, this would correspond to a dose of 43. A homemade mask worn by a susceptible person would have reduced infection probability to 16%, a surgical mask with bad fit to 8%, an N95 with a bad fit to 1%, and an N95 with a good fit to negligible. If the infective person wore a homemade mask, then, using our earlier assumptions, we would end up with the respective infection probabilities being 10% for susceptible persons with homemade masks, 5% for those with surgical masks, and negligible for N95 masks, with or without a poor fit. The risk in other parts of the plane would have been even smaller.

This gives a rough idea of what we can expect with COVID-19. One caveat is that with the much larger number of cases, we can see more lextreme events. It is typical of fat-tailed distributions to throw up unpleasant surprises. A superspreader could drive an extreme event with an immense magnitude of virus shedding beyond our estimates.

Another risk factor arises from the non-linearity in the dose-response relationship. This risk has been shown in multiple flights where everyone wore masks. For example, at least one case of inflight transmission occurred on a flight where everyone wore a mask (Eldin et al., 2020). The type of masks worn is not known. If we assume homemade masks, then we note that they don't eliminate all risk. In addition, there were a couple of errors of educational value made on the flight. One was that people took off their masks to eat. The other is that one person who ended up infected wore his mask only partially in order to talk to his family for quite some time. In a high-risk environment, non-use of masks for some time can increase infection risk substantially.

Let us consider a hypothetical case motivated by the above incident. There, the infected person happened to be close to four infective persons for a significant time. A susceptible person surrounded by four infective persons would have experienced a dose of 172 with an infection probability 82% without masks on anyone if we assume the same parameters as in the SARS super-spreading example mentioned above. Now, let us assume that the infective persons wore handmade masks and the susceptible an N95 with a bad fit. The dose would be 3.3, with infection probability around 3%. Let us now consider a situation where the susceptible and infective persons wore masks only half the time, and removed them simultaneously. The dose experienced would be the average of the above two, 87.65. The infection probability associated with this dose is 58%. This is much higher than the average of the above two infection probabilities of around 43%. Non-linear functions have no sense of justice. **In a high-risk environment, the consequences of small errors can be magnified.**

5   Conclusions

In summary, mask use by everyone would reduce the infection risk on planes and other close-contact confined places substantially, but not eliminate them. The use of N95 masks can eliminate almost all risk. To be safe, we need to keep the mask on all the time in a high-risk environment. However, the fat-"tail can wag the dog" (Cirillo and Taleb, 2020), and we might incur some residual risk that we have not modeled.

Acknowledgments

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